100. On q-Spaces

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1. Introduction. The notion of q-spaces has been introduced by E. Michael [4]. A topological space Y is called a q-space if every $y \in Y$ has a sequence $\{N_i\}$ of neighborhoods satisfying the following condition (q):

$$\text{(q)} \quad \begin{cases} \text{If } \{y_i\} \text{ is an infinite sequence of points in } Y \text{ such that} \\ y_i \in N_i \text{ for each } i, \text{ then } \{y_i\} \text{ has an accumulation point} \\ \text{in } Y. \end{cases}$$

An M-space is clearly a q-space, but the converse does not hold. Indeed, a locally compact space is a q-space but not always an M-space (cf. K. Morita [5]). These two notions of spaces, however, are connected by the notion of almost open mappings due to P. Vopěnka [6]. Namely we shall prove

Theorem 1.*) *A regular space Y is a q-space if and only if there exists an almost open continuous mapping f from a regular M-space onto Y.*

The definition of almost open mapping is as follows: A mapping $f: X \to Y$ is called almost open if for any point y of Y there is a point $x \in f^{-1}(y)$ having a basis of open sets such that the image of each member of the basis is open in Y. This notion of mappings is also available for a characterization of pointwise countable type in the sense of A. Arhangel'skii [1] by paracompact M-spaces.

Theorem 2. *A topological space Y is of pointwise countable type if and only if there exists an almost open continuous mapping f from a paracompact M-space X onto Y.*

Recently it has been shown by T. Isiwata [3] that the product of M-spaces need not be an M-space. By a counter example given by him, we see also that the product of q-spaces is not necessarily a q-space. In the final section we shall give a sufficient condition for the product of q-spaces to be a q-space.

2. Proof of Theorem 1. Let Y be a regular q-space and let y be a point of Y. Then y has a sequence $\{N_i\}$ satisfying condition (q) and $\bar{N}_{i+1} \subset N_i$ for each i. Let us put $K = \cap \{\bar{N}_i : i = 1, 2, \cdots\}$. Then each sequence in K has an accumulation point in K, since $\{N_i\}$ satisfies

*) Cf. J. Nagata: Mappings and M-spaces, Proc. Japan Acad., **45**, 140–144 (1969), which appeared after the preparation of this paper.