99. *Propagation of Chaos for Certain Markov Processes of Jump Type with Nonlinear Generators. I*

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1. Introduction. Let Q be a set endowed with a σ-field $\mathcal{F}$ of its subsets such that each single point set $\{x\}$ is in $\mathcal{F}$, and denote by $\mathcal{P}$ the set of probability measures on $(Q, \mathcal{F})$. Suppose we are given a kernel $A_f(x, \Gamma)$ indexed by $f \in \mathcal{P}$ with the form:

$$A_f(x, \Gamma)=\sum_{n=1}^{\infty}\int\cdots\int A_n^{(x_1,\cdots,x_n)}(x, \Gamma)f(dx_1)\cdots f(dx_n), \quad \Gamma \in \mathcal{F},$$

and assume that the following 3 conditions are satisfied.

(i) For each $n\geq 1$, $x, x_1, \cdots, x_n \in Q$, $A_n^{(x_1,\cdots,x_n)}(x, \cdot)$ is a bounded signed measure on $(Q, \mathcal{F})$ which is nonnegative outside $\{x\}$ and has zero total mass.

(ii) For each $n\geqslant 1$, $\Gamma \in \mathcal{F}$, $A_n^{(x_1,\cdots,x_n)}(x, \Gamma)$ is a measurable function of $(x, x_1, \cdots, x_n)$, symmetric in $(x_1, \cdots, x_n)$ when x is fixed, and $A_n^{(x_1,\cdots,x_n)}(x, \{x\})$ is measurable in $(x, x_1, \cdots, x_n)$.

(iii) $q=\sum_{n=1}^{\infty} q_n<\infty$, where $q_n=\sup_{x,x_1,\cdots,x_n\in Q} A_n^{(x_1,\cdots,x_n)}(x, Q-\{x\})$.

We are concerned with the following nonlinear equation:

$$\frac{du(t)}{dt}=Au(t), \qquad u(0+)=f, \tag{1.1}$$

where the initial value f and the solution u, for each $t>0$, are in $\mathcal{P}$, and $(Au)(\cdot)=\int_Q A_u(x, \cdot)u(dx)$.

Denote by Q^n $(\mathcal{F}^n)$ the n-fold product space $Q\times\cdots\times Q$ (σ-field $\mathcal{F}\times\cdots\times\mathcal{F}$) of Q $(\mathcal{F})$, and let A_n be a linear operator from the space $\mathfrak{M}_n$ of bounded signed measures on $(Q^n, \mathcal{F}^n)$ into itself defined by

$$(A_n u)(\Gamma)=\int_{Q^n} u(dx_1\cdots dx_n)\sum_{N=1}^{n-1} n^{-N} \sum_{i,i_1,\cdots,i_N}^{(n)}\int_Q A_N^{(x_{i_1},\cdots x_{i_N})}(x_i, \Gamma),$$

where $\sum_{i,i_1,\cdots,i_N}^{(n)}$ is the sum with respect to all $(i, i_1, \cdots, i_N)$ such that $i, i_1, \cdots, i_N$ are all different and $1\leq i, i_1, \cdots, i_N\leq n$; χ_Γ is the indicator function of $\Gamma \in \mathcal{F}^n$, and the notation $A_N^{(x_{i_1},\cdots,x_{i_N})}(x_i, \varphi)$ for $\varphi=\varphi(x_1, \cdots \cdots, x_n)$ stands for

$$\int_Q A_N^{(x_{i_1},\cdots,x_{i_N})}(x_i, dx)\varphi(\cdots, x_{i-1}, x, x_{i+1}, \cdots).$$

Consider the linear equation for $n=2, 3, \cdots$: