93. *A Remark on a Conjecture of Paley*

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The standard symbols of the Nevanlinna theory

$$\log^+,\ M(r,f),\ m(r,a),\ N(r,a),\ T(r,f),\ \delta(a,f)$$

are used throughout this note. We define

$$N(r)=N(r,0)+N(r,\infty)$$

and

$$K(f)=\limsup_{r\to\infty}\frac{N(r)}{T(r)}.$$

Paley [3] conjectured that an integral function of finite order $\rho>\frac{1}{2}$ satisfies

$$\limsup_{r\to\infty}\frac{m(r,f)}{\log M(r,f)}\geqq\frac{1}{\pi\rho}.$$

The object of the present note is to show that as an *immediate consequence* of Edrei-Fuchs's results [1, 2] we obtain

Theorem. *If an integral function of finite order $\rho>\frac{1}{2}$ satisfies*

$$\sum_{a\neq\infty}\delta(a,f)=1, \tag{1}$$

then we have

$$\frac{1}{2}\geqq\limsup_{r\to\infty}\frac{m(r,f)}{\log M(r,f)}\geqq\liminf_{r\to\infty}\frac{m(r,f)}{\log M(r,f)}\geqq\frac{1}{\pi}.$$

In particular if there exists a finite a with $\delta(a,f)=1$, then

$$\lim_{r\to\infty}\frac{m(r,f)}{\log M(r,f)}=\frac{1}{\pi}. \tag{2}$$

Edrei and Fuchs proved the following theorem and lemmas.

Theorem A [1]. *If the integral function $f(z)$ in question satisfies* (1), *then*

$$\lim_{r\to\infty}\frac{T(r,f')}{T(r,f)}=1,\ K(f')=0,$$

and $f(z)$ is necessarily of positive integral order and of regular growth.

Lemmas [2]. *Let $f(z)$ be a meromorphic function of finite lower order μ and p be the non-negative integer defined by the inequalities*

$$p-\frac{1}{2}\leqq\mu<p+\frac{1}{2}.$$

Let $E(u,p)$ be the primary factor of genus p. Now suppose that the function $f(z)$ satisfies