92. *Angular Cluster Sets and Oricyclic Cluster Sets*

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1. Let G be the unit disk $|z|<1$ and Γ be its circumference $|z|=1$. For a point $\zeta\in\Gamma$, let $V=V(\zeta)$ be an angle with vertex at ζ and $K=K(\zeta)$ be an inscribed disk at ζ, that is,

$$K(\zeta)=\{z\,;\,|z-\rho\zeta|<1-\rho\},$$

where ρ is a constant, $0<\rho<1$.

For a function $f(z)$ given in G, we set

$$C(\zeta, K)=C(\zeta, K, f)$$
$$=\{a\,;\text{ there is a sequence } z_\nu\in K(\zeta),\ z_\nu\to\zeta,\ f(z_\nu)\to a\}.$$

$C(\zeta, V)=C(\zeta, V, f)$ is defined similarly.

We put

$$C_{\mathfrak{A}}(\zeta, f)=\bigcup_V C(\zeta, V, f),\quad C_{\mathfrak{O}}(\zeta, f)=\bigcap_K C(\zeta, K, f),$$

where summation and intersection are taken over all $V(\zeta)$ and $K(\zeta)$. $C_{\mathfrak{A}}$ and $C_{\mathfrak{O}}$ are called *angular cluster set* and *oricylic cluster set*, respectively [2].

Obviously $C_{\mathfrak{A}}\subset C_{\mathfrak{O}}$. We will show here that $C_{\mathfrak{A}}(\zeta, f)=C_{\mathfrak{O}}(\zeta, f)$ except on a set of σ-porosity of the order $1/2$ (see the definition below), for any arbitrary function $f(z)$.

If $C_{\mathfrak{F}}(\zeta, f)$ is the fine cluster set at ζ [4], Brelot and Doob [4] proved that $C_{\mathfrak{A}}(\zeta, f)\subset C_{\mathfrak{F}}(\zeta, f)$ for harmonic or holomorphic $f(z)$. Since $K(\zeta)$ is a fine neighborhood of ζ, we have $C_{\mathfrak{A}}\subset C_{\mathfrak{F}}\subset C_{\mathfrak{O}}$. Thus the relation between $C_{\mathfrak{A}}$ and $C_{\mathfrak{O}}$ will suggest some relation between $C_{\mathfrak{A}}$ and $C_{\mathfrak{F}}$.

2. Let us define some notions. A *KK* (or *VV*)-*singular point* is the point $\zeta\in\Gamma$ such that $C(\zeta, K', f)\neq C(\zeta, K'', f)$ (or $C(\zeta, V', f)\neq C(\zeta, V'', f)$) for some pair of inscribed disks $K'(\zeta)$ and $K''(\zeta)$ (or angles $V'(\zeta)$ and $V''(\zeta)$). The set of all *KK* (or *VV*)-singular points is called *KK* (or *VV*)-*singular set* and denoted by $E_{KK}(f)$ (or $E_{VV}(f)$).

A *GK* (or *GV*)-singular point is the point $\zeta\in\Gamma$ such that $C(\zeta, K, f)\neq C(\zeta, f)$ (or $C(\zeta, V, f)\neq C(\zeta, f)$) for some $K(\zeta)$ (or $V(\zeta)$), where $C(\zeta, f)$ is the cluster set at ζ, that is,

$$C(\zeta, f)=\{a\,;\text{ there is a sequence } z_\nu\in G,\ z_\nu\to\zeta,\ f(z_\nu)\to a\}.$$

GK (or *GV*)-singular set is denoted by $E_{GK}(f)$ (or $E_{GV}(f)$).

KV-singularity is defined analogously.

For a $\varepsilon>0$, we set $U_\varepsilon(\zeta)=\{z\,;\,|z-\zeta|<\varepsilon\}$ (ε-neighborhood). Sup-