135. *The Subordination of Lévy System for Markov Processes*

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§1. **Preliminary notions and the result.** For each process $x(t)$ belonging to a certain class of Markov processes, the Lévy measure $n(x, dy)$ is defined as follows [1]:

$$(1) \qquad \lim_{t\to+0} T_t f(x)/t = \lim_{t\to+0}\int_S f(y)P(t, x, dy)/t = \int_S f(y)n(x, dy) \qquad \text{for every } x\in D$$

where S and $\hat{S}$, are respectively, a locally compact Hausdorff space satisfying the 2nd axiom of countability and its one-point compactification, D is a bounded open set in S, and f is a function in $C(\hat{S})$ whose support does not intersect D. $\{T_t\}$ and $\{P(t, x, dy)\}$ respectively, are the semigroup and the transition functions of the process $x(t)$, and the convergence in (1) is a bounded convergence in D.

We know that, when the time of such a Markov process is changed by a temporally homogeneous non-decreasing Lévy process $h(t)$ which is independent of $x(t)$ and has the Lévy measure $\dot{n}(t)$:

$$(2) \qquad \dot{E}e^{rh(t)} = \exp\left[-t\left\{cr + \int_0^\infty (1-e^{-ru})\dot{n}(du)\right\}\right]$$

$$c \geq 0, \quad \int_0^\infty \frac{u}{1+u}\dot{n}(du) < \infty,$$

then the Lévy measure $\tilde{n}(x, dy)$ of the new Markov process is as follows [1]:

$$(3) \qquad \tilde{n}(x, dy) = cn(x, dy) + \int_0^\infty P(t, x, dy)\dot{n}(dt).$$

Furthermore, for each process $x(t)$ belonging to a wider class of Markov processes, that is, the class of Hunt processes with reference measures on S, the Lévy system $(n(x, dy), A)$, the pair of a kernel $n(x, dy)$ and an additive functional $A(t)$ of $x(t)$, is defined as a generalization of the Lévy measure defined above as follows [2]:

$$(4) \qquad E_x \sum_{s\leq t} f(x(s-), x(s)) = E_x\left[\int_0^t \left\{\int_{\hat{S}} f(x(s), y)\, n(x(s), dy)\right\} dA(s)\right]$$

where f is an $F(S\times\hat{S})$-measurable non-negative function such that $f(x, x)=0$ for any $x\in S$, and $F(S\times\hat{S})$ is the completion of the topological Borel field on $S\times\hat{S}$ with respect to the family of all bounded measures. If $A(t)$ is the minimum of t and the life time of $x(t)$, then