133. *On Conservativity of Algebraic Function Fields*

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1. Let K be a field of algebraic functions of one variable over a field k of characteristic $p \neq 0$. Throughout this note, we assume that K is separable over k and k is algebraically closed in K. If the genus of K/k is invariant under any constant field extension of K/k, we say that K/k is *conservative*. E. Artin has proved that K/k is conservative if and only if for all finite purely inseparable constant field extensions $\tilde{K}/\tilde{k}$ of K/k, the genus of K/k is equal to the genus of $\tilde{K}/\tilde{k}$ (Chapter 15 of [1]).

Let K/k be as above, $M=\bigcup_{i=1}^{n} M_i$ a complete normal model of K/k, where $M_1, \cdots, M_n$ are affine models defined by affine k-algebras $A_1, \cdots, A_n$ respectively. Furthermore, we assume that each A_i is isomorphic to $k[X_{i1}, \cdots, X_{il_i}]/\mathfrak{a}_i$, where $k[X_{i1}, \cdots, X_{il_i}]$ is a polynomial ring and $\mathfrak{a}_i$ is a prime ideal of $k[X_{i1}, \cdots, X_{il_i}]$. In this note, we fix a normal complete model M and a set of equations for M, i.e., the union $\bigcup_{i=1}^{n} B_i$ where $B_i=\{F_{i1}(X), \cdots, F_{is_i}(X)\}$ is a basis of $\mathfrak{a}_i$. Let Ω be the set of all coefficients in the equations belonging to the set of equations for M, $\Delta=\{a_1, a_2, \cdots, a_m\}$ a p-basis of $k^p(\Omega)$ over k^p and let $\Delta^{p^{-1}}=\{a_1^{p^{-1}}, a_2^{p^{-1}}, \cdots, a_m^{p^{-1}}\}$. Then we have the following:

Theorem. *K/k is conservative if and only if the genus of K/k is equal to the genus of $K(\Delta^{p^{-1}})/k(\Delta^{p^{-1}})$.*

Remark. (1) We say that an algebraic function field $\tilde{K}/\tilde{k}$ is a constant field extension of K/k if $\tilde{K}=\tilde{k}K$ and K is free from $\tilde{k}$ over k. If we choose the above $a_i^{p^{-1}} (i=1, 2, \cdots, m)$ from a fixed complete field k^* which contains k, then we can construct the constant field extension $K(\Delta^{p^{-1}})/k(\Delta^{p^{-1}})$ of K/k by the method of Chevalley [2].

(2) Let M and A_i $(i=1, 2, \cdots, m)$ be as stated above. Then the model of $K(\Delta^{p^{-1}})/k(\Delta^{p^{-1}})$ defined by $k(\Delta^{p^{-1}})[A_i]$ $(i=1, 2, \cdots, n)$ is denoted by $M\otimes k(\Delta^{p^{-1}})$ (to prove Theorem, we shall consider this model $M\otimes k(\Delta^{p^{-1}})$ as a model over k). The geometric genus of M (resp. $M\otimes k(\Delta^{p^{-1}})$) is equal to the genus of K/k (resp. $K(\Delta^{p^{-1}})/k(\Delta^{p^{-1}})$) (cf. §6 of [4]).

(3) By a differential constant field for M (or K/k), we mean a field k_0 which satisfies the following three conditions: