# 131. *Structure Theorems for Some Classes of Operators*

By I. ISTRĂTESCU
Politechnic Institute Timişoara Roumania

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**1.** We consider bounded linear operators on a Hilbert space $H$. Denote by $\sigma(T)$, $\sigma_p(T)$, $\sigma_r(T)$, $\sigma_c(T)$ the spectrum, the point spectrum, the residual spectrum and the continuous spectrum respectively, by $W(T)=\{(Tx, x): \|x\|=1\}$ the numerical range. It is known [3] that $W(T)$ is convex and conv $\sigma(T)\subseteq$ cl $W(T)$ (conv = convex hull, cl = closure). An operator $T$ is said to be hyponormal if $T^*T-TT^*\geq 0$, or equivalently if $\|T^*x\|\leq\|Tx\|$ for every $x\in H$. As in [1] an operator is said to be restriction-convexoid (reduction-convexoid) if the restriction of $T$ to every invariant (invariant under $T$ and $T^*$) subspace is convexoid, where convexoid means that conv $\sigma(T)=$ cl $W(T)$.

In this Note we give some theorems on structure of hyponormal and restriction-convexoid operators whose spectrum lies on a convex curve.

**2.** Our main result in this section is

**Theorem 1.** *If $T$ is a hyponormal operator and has the following properties*

1° $T^p=ST^{*p}S^{-1}+C$ *for some* $S$ *for which* $0\notin \text{cl}\, W(S)$ *and* $C=$ *compact operator*

2° *if* $\mu, \lambda\in\sigma(T)$, $1+\dfrac{\lambda}{\bar\mu}+\left(\dfrac{\lambda}{\bar\mu}\right)^2+\cdots+\left(\dfrac{\lambda}{\bar\mu}\right)^{p-1}\neq 0$

*then $T$ is a normal operator.*

For the proof we need the following

**Lemma 1.** *If $T$ is a hyponormal operator which is the sum of a self-adjoint operator $A$ and a compact operator $C$, then $T$ is a normal operator.*

**Proof.** We denote by $\sigma_r^*(T)$ the set of complex numbers $\lambda$ such that $T-\lambda I$ has a continuous inverse and that the range of $T-\lambda I$ is not dense in $H$ and $\sigma_c^*(T)$ is the set of complex numbers $\lambda$ which does not belong to $\sigma_p(T)$ and for which there exists a sequence $\{x_n\}$ of unit vectors in $H$ such that $\|Tx_n-\lambda x_n\|\to 0$ as $n\to\infty$.

Since $T$ is hyponormal it is known that $T$ can be expressed uniquely as a direct sum $T=T_1\oplus T_2$ defined on a product space $H=H_1\oplus H_2$ where $H_1$ is spanned by all the proper vectors of $T$ such that: (a) $T_1$ is normal and $\sigma(T_1)=$ cl $\sigma_p(T)$, (b) $T_2$ is hyponormal and $\sigma_p(T_2)=\varnothing$, (c) $T$ is normal if and only if $T_2$ is normal.