129. On the Category of $L^1(G)\cap L^p(G)$ in $A^q(G)$*)

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1. Introduction and the main results.

Let G and $\hat{G}$ be two locally compact abelian groups in Pontrjagin duality. The integration with respect to a suitably normalized Haar measure on G is indicated by the expressions such as

$$\int_G f(x)\,dx \tag{1}$$

Let $C_c(G)$ denote the space of all continuous complex-valued functions on G each of which vanishes outside of some compact set, and $C_0(G)$ the set of continuous functions each of which vanishes at infinity. We shall denote $A^p(G)$ $(1\leq p<\infty)$ the space of functions f in $L^1(G)$ whose Fourier transforms $\hat{f}$ belong to $L^p(\hat{G})$ $(p\geq 1)$ and with the norm defined by

$$\|f\|^p=\|f\|_1+\|\hat{f}\|_p \tag{2}$$

where $\|f\|_1=\int_G |f(x)|\,dx$ and $\|\hat{f}\|_p=\left(\int_{\hat{G}} |\hat{f}(\hat{x})|^p\,d\hat{x}\right)^{1/p}$, $d\hat{x}$ denotes the integration with respect to Haar measure on $\hat{G}$. Clearly, $A^p(G)$ is a dense ideal in $L^1(G)$ and is a Banach algebra under convolution with the norm $\|\cdot\|^p$ (see Larsen, Liu and Wang [6]).

We denote T_1 and T_2 the Fourier transforms on $L^1(G)$ and $L^2(G)$ respectively. That is

$$T_1f(\hat{x})=\int_G (-x,\hat{x})f(x)\,dx \tag{3}$$

and

$$\|T_1f\|_\infty\leq\|f\|_1 \tag{4}$$
$$\|T_2f\|_2=\|f\|_2.$$

If $f\in C_c(G)$, the Fourier transform T is defined by the usual expression

$$Tf(\hat{x})=\int_G (-x,\hat{x})f(x)\,dx, \tag{5}$$

and $T_1f=T_2f=Tf$ for every $f\in C_c(G)$. Throughout this present note, we suppose essentially that $1<p<2$ and $1/p+1/q=1$. A. Weil [9; pp. 116–117] has shown, by using the convexity theorem of M. Riesz

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