128. *On Some Properties of $A^p(G)$-algebras*[*)]

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1. Introduction. Let G be a locally compact abelian group with dual group $\hat{G}$. We denote dx and $d\hat{x}$ the Haar measures of G and $\hat{G}$ respectively. Recently, Larsen, Liu, and Wang [4] have investigated a space $A^p(G)$ consisting of all complex-valued functions $f \in L^1(G)$ whose Fourier transforms $\hat{f}$ belong to $L^p(\hat{G})$ $(p \geqslant 1)$. In this paper, we shall show further investigations of the algebra $A^p(G)$ proving the existence of the approximate identities of $A^p(G)$ and using the approximate identity to give a reproof of Theorem 5 in [4]. We show also that the closed primary ideal of $A^p(G)$ is maximal.

2. The approximate identities of $A^p(G)$-algebras. It is clear that $A^p(G)$ is an ideal dense in $L^1(G)$ under convolution. Indeed, for any $f \in A^p(G)$ and $g \in L^1(G)$,

$$\| \widehat{f*g} \|_p \leqslant \| \hat{g} \|_\infty \| \hat{f} \|_p$$

proving $f*g \in A^p(G)$ and the density of $A^p(G)$ in $L^1(G)$ follows from the fact that if $\{e_\alpha\}$ is an approximate identity in $L^1(G)$ whose Fourier transforms have compact supports then $e_\alpha \in A^p(G)$ and for an arbitrary function $f \in L^1(G)$ we have

$$f*e_\alpha \in A^p(G) \text{ and } \| f*e_\alpha - f \|_1 \to 0.$$

Define the norm of $f \in A^p(G)$ $(1 \leqslant p < \infty)$ by

$$\| f \|^p = \| f \|_1 + \| \hat{f} \|_p$$

where $\| f \|_1 = \int_G | f(x) | \, dx$ and $\| \hat{f} \|_p = \left(\int_{\hat{G}} | \hat{f}(\hat{x}) |^p \, d\hat{x} \right)^{1/p}$. Then $A^p(G)$ is a commutative Banach algebra under convolution as its product and with the norm $\| \cdot \|^p$ (see [4; Theorem 3]).

We say here an approximate identity for $A^p(G)$ a family $\{e_\alpha\}$ of functions in $A^p(G)$ such that for any $f \in A^p(G)$ and $\varepsilon > 0$, there exists $e_\alpha \in \{e_\alpha\}$ such that $\| e_\alpha * f - f \|^p < \varepsilon$.

Theorem 1. *The Banach algebra $A^p(G)$ has an approximate identity with the properties that it is also the bounded approximate identity for $L^1(G)$ and whose Fourier transform has compact support in $\hat{G}$.*

Proof. By Rudin [7] Theorem 2.6.6, we see that there is a

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