123. *Korteweg-deVries Equation. II*

Finite Difference Approximation

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In the preceding note [1] we announce the fact that the Cauchy problem for the KdV equation has global smooth solution uniquely for any sufficiently smooth initial data. Here we show the existence of the computable approximate solutions of the Cauchy problem for the KdV equation by the method of finite difference schemes.

To approximate the Cauchy problem for the KdV equation

$$(1) \qquad D_t u = uDu + D^3 u \qquad (x, t) \in R^1 \times (0, \infty) \qquad u = u(x, t)$$

$$(2) \qquad u(x, 0) = f(x) \qquad x \in R^1 \qquad D_t = \frac{\partial}{\partial t}, \quad D = \frac{\partial}{\partial x}$$

We propose following special implicit finite difference scheme

$$(3) \qquad D^+ u_j^n = E u_j^{n+1} D_0 u_j^{n+1} + D_+ D_-^2 u_j^{n+1} \qquad j = 0, \pm 1, \pm 2, \cdots \quad n = 0, 1, 2, \cdots$$

$$(4) \qquad u_j^0 = f_j \qquad j = 0, \pm 1, \pm 2, \cdots$$

Here we use the following notations

$$u_j^n = u(jh, n\Delta t) \quad h: \text{mesh-width} \quad \Delta t = \lambda h^3: \text{time-step}$$
$$\lambda = \text{const.}: \text{mesh ratio}$$

$$D^+ u_j^n = \frac{1}{\Delta t}(u_j^{n+1} - u_j^n)$$

$$D_+ u_j^n = \frac{1}{h}(u_{j+1}^n - u_j^n)$$

$$D_- u_j^n = \frac{1}{h}(u_j^n - u_{j-1}^n)$$

$$D_0 u_j^n = \frac{1}{2h}(u_{j+1}^n - u_{j-1}^n)$$

$$E u_j^n = \frac{1}{3}(u_{j+1}^n + u_j^n + u_{j-1}^n)$$

To solve (1) with respect to u_j^{n+1} we use following iteration

$$(5) \qquad u_j^{n+1-\frac{1}{m+1}} = u_j^n + \lambda h^2 E u_j^{n+1-\frac{1}{m}} F u_j^{n+1-\frac{1}{m}} + 8\lambda G u_j^{n+1-\frac{1}{m}} \qquad m = 1, 2, 3, \cdots$$

Here

$$F u_j^n = \frac{1}{2}(u_{j+1}^n - u_{j-1}^n)$$

$$G u_j^n = \frac{1}{8}(u_{j+1}^n - 3u_j^n + 3u_{j-1}^n - u_{j-2}^n)$$