121. *A Remark on Singular Integral Operators and Reflection Principle for Some Mixed Problems*

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§ 1. Introduction. In this note we consider at first the singular integral operators whose symbols are not necessarily smooth along some hypersurfaces in R^n (Theorem 1). We apply its results to some hyperbolic mixed problems. And we show how to derive the finiteness of propagation speed for these problems. As one might observe, our method of the proof of Theorem 1 is due essentially to A.P. Carderon-A. Zygmund, but the obtained results can be applied directly to some mixed problems.

As for mixed problems, S. Mizohata [4] treated some hyperbolic equations of higher order, and the author [3] showed an extention to the case of fourth order, imposing the assumption only on the boundary. At that time this assumptions clarified the type of the equations suitable to the boundary conditions imposed in [4]. Then K. Asano and T. Shirota [1] investigated the singular integral operators attached to the same type boundary conditions in a half space, and treated the equation (E) below. Now let us remark that Holmgren's transformations (5.1) at the boundary of the equation in [3] and [4] yield such a type of equations as (E), and (E) is the closed form with respect to (5.1). By virtue of Theorem 1 we can apply the reflection principle to the equation (E). This principle makes the treatment in [1] fairly simple. Finally we shall show the finiteness of the propagation speed of the solution, using Lemmas 2, 3 (Theorem 2). The detailed proof will be given in a forthcoming paper.

§2. Singular integral operator. Hereafter we follow the notation of [2]. First of all, let us define the following class of functions.

Definition. A function $h(x)$ defined in R^n is said to be piecewise in $\mathscr{B}^{1+\alpha}$ relative to given hypersurfaces S, if $h(x)$ has the properties: (i) $h(x)$ is continuous in R^n. (ii) $h(x)$ is in $C^{1+\alpha}(\bar{\omega})$, where ω is any connected component of R^n-S. $(0<\alpha<1)$

Theorem 1. *Assume that $h(x, \xi)$ defined in $R^n\times(R^n-\{0\})$ be a C^∞ function of homogeneous degree zero and be piecewise in $\mathscr{B}^{1+\alpha}$ relative to hypersurfaces S with respect to x. Then, for the singular integral operators H, H_1 and H_2 with such symbols $h(x, \xi)$, $h_1(x, \xi)$ and $h_2(x, \xi)$ respectively, we have the following facts:* $H\Lambda-\Lambda H$, $H^*\Lambda-\Lambda H^*$,