159. *Products of M-Spaces*

By Takesi ISIWATA

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The spaces considered here are always completely regular T_1-spaces and mappings are continuous. We have showed in the previous paper [4] that the product of M-spaces need not be an M-space. In this paper, introducing a new class $\mathfrak{C}(M)$ of M-spaces, we shall prove in § 2 the following main theorem:

$X \in \mathfrak{C}(M)$ if and only if the product $X \times Y$ is an M-space for every M-space Y.

In § 3, we shall show that $\mathfrak{C}(M)$ contains the class $\mathfrak{C}(*)$ which contains all M-spaces X such that X satisfies one of the following conditions: (a) X satisfies the first axiom of countability, (b) X is locally compact and (c)X is paracompact (see [2], p. 897), moreover $\mathfrak{C}(M)$ contains the class $\mathfrak{C}(C)$ (§ 1 below).

§1. Definitions and preliminaries. A space X is called an *M-space*, the notion of which is introduced by K. Morita [6], if there exists a normal sequence $\{\mathcal{U}_i\}$ of open coverings of X satisfying the following condition (M):

(M) If $\{K_i\}$ is a sequence of non-empty closed subsets of X such that $K_{i+1} \subset K_i$ and $K_i \subset \mathrm{St}(x_0, \mathcal{U}_i)$ for each i and some fixed point $x_0 \in X$, then $\cap K_i \neq \emptyset$.

In the following we call $\{\mathcal{U}_i\}$ mentioned above, for simplicity, an *M-normal sequence* of X. A sequence $\{x_i\}$ in X is said to be an *(M)-sequence* if $x_i \in \mathrm{St}(x_0, \mathcal{U}_i)$ for every i and some fixed point x_0 of X and some M-normal sequence $\{\mathcal{U}_i\}$ of X. In [2] the class $\mathfrak{C}(*)$ has been introduced as the set of all M-spaces satisfying the following condition (*):

(*) Any (M)-sequence has a subsequence whose closure is compact.

The symbol $\mathfrak{C}(C)$ denotes the class of all spaces P such that the product $P \times Q$ is countably compact for every countably compact space Q. This class has been introduced by Frolík [1] and it is obvious that $P \in \mathfrak{C}(C)$ implies that $F \in \mathfrak{C}(C)$ for every closed subset F of P. We shall consider the class $\mathfrak{C}(M)$ consisting of all M-spaces X satisfying the following condition (CM):

(CM) For any discrete subsequence N of any (M)-sequence of X and for any non-empty subset S of $K - X$ where K is any compactification of X, the subspace $N \cup S$ of K is not countably compact.