158. On the Bi-ideals in Semigroups

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Let S be a semigroup, and A be a non-empty subset of S. We shall say that A is a *bi-ideal* or *(1, 1)-ideal* of S if the following conditions hold:

(i) A is a subsemigroup of S.

(ii) $ASA \subseteq A$.

The notion of bi-ideal was introduced by R. A. Good and D. R. Hughes [2]. It is also a special case of the (m, n)-ideal introduced by the author [4].

In this short note we give a summary of some results concerning the bi-ideals of semigroups, and we announce some new results. For the terminology not defined here we refer to the books by A. H. Clifford and G. B. Preston [1]. Proofs of the results will not be given.

Theorem 1. *Let S be an arbitrary semigroup. Then any left (right, two-sided, and quasi-) ideal of S is a bi-ideal of S.*

Theorem 2. *Suppose that $A_1, \cdots, A_n$ are bi-ideals of a semigroup S. Then the intersection $B=\bigcap_{i=1}^{n} A_i$ either is empty or it is a bi-ideal of S.*

We say that a bi-ideal A of a semigroup S is a *proper bi-ideal* of S if A is a proper subset of S, that is, the set $S-A$ is not empty. It is easy to see that a group has not proper bi-ideals, and what is more this property characterizes the class of groups among semigroups.

Theorem 3. *A semigroup S is a group if and only if it has not proper bi-ideals.*

By a bi-ideal of a semigroup S generated by a non-empty subset A of S we mean the smallest bi-ideal of S containing A. Let us denote this bi-ideal by $(A)_{(1,1)}$. If the set A consists of a single element then the bi-ideal of S generated by A is said to be a *principal bi-ideal* of S. It is easy to show that the following assertion is true.

Theorem 4. *Let a be an arbitrary element, and A be a non-empty subset of S. Then $(A)_{(1,1)}=A \cup A^2 \cup ASA$ and $(a)_{(1,1)}=a \cup a^2 \cup aSa$.*

An important property of the bi-ideals is formulated in the following theorem. This was proved by the author (see [6], first part).

Theorem 5. *Let A be a bi-ideal and B be a non-empty subset of S. Then the products AB and BA are bi-ideals of S.*