157. *Mixed Problems for Degenerate Hyperbolic Equations of Second Order*

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1. Introduction. In this note we shall deal with the following equation:

$$u_{tt}=p(x)u_{xx}+f(x,t) \tag{1.1}$$

in $R^1_+\times(0,\infty)$, where $p(x)$ is a real valued function such that;

(i) $p(x)\in C^0(\bar{R}^1_+)$ and $0\leqq p(x)$ ($p(x)$ never vanishes except at $x=0$)

(ii) for $x\to\infty$, $p(x)$ remains bounded, and moreover bounded away from zero

(iii) $p(x)^{-1}$ is summable in the neighborhood of the origin.

Our boundary conditions are as follows:

Case I $u=0$ at $x=0$

Case II $u_x+hu=0$ at $x=0$ (h is a real number).

Since (1.1) is not strictly hyperbolic, we might not expect the mixed problems with above boundary conditions be L^2-well-posed, but we can show that our problem is well suited on a certain function (Hilbert) space.

2. Function spaces $L^2(R^1_+, p^{-1})$ and $H^2(R^1_+, p)$. In this section we establish two function spaces in which we develop our arguments.

Definition 2.1. A distribution $u(x)$ on R^1_+ is said to be in $L^2(R^1_+, p^{-1})$, if and only if

$$\|u\|^2_{p^{-1}}=\int_0^\infty |u|^2p^{-1}dx \tag{2.1}$$

is finite.

Definition 2.2. A distribution $u(x)$ on R^1_+ is said to be in $H^2(R^1_+, p)$, if and only if

$$\|u\|^2_{2,p}=\int_0^\infty(|u|^2+p(x)|u_{xx}|^2)dx \tag{2.2}$$

is finite.

Lemma 2.3. *If $u(x)$ belongs to $H^2(R^1_+, p)$, then $u_x(0)=\lim u_x(x)$ exists and*

$$|u_x(0)|^2\leqslant\varepsilon\int_0^\infty p(x)|u_{xx}|^2\,dx+C(\varepsilon)\int_0^\infty|u|^2\,dx \tag{2.3}$$

is valid for any positive ε.

Lemma 2.4. *If $u(x)$ is in $H^2(R^1_+, p)$, then $u(x)$ is in $H^1(R^1_+)$ and*

$$\int_0^\infty|u_x|^2\,dx\leqslant\varepsilon\int_0^\infty p(x)|u_{xx}|^2\,dx+C(\varepsilon)\int_0^\infty|u|^2\,dx \tag{2.4}$$