156. *Some Remarks on Radiation Conditions*

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(Comm. by Kinjirô KUNUGI, M. J. A., Oct. 13, 1969)

Introduction. In linear wave propagation problems which are time-independent and which take place in unbounded domain, it is in general not possible to characterize the solutions having the desired physical characteristics by imposing only boundedness conditions at infinity. To do so it is necessary to impose sharper conditions at infinity called the radiation conditions.

The radiation conditions have been discovered by Sommerfeld [5] for the reduced acoustic equation (Helmholtz equation) and by Silver [4] and, independently, by Müller [3] for the reduced Maxwell equations (vector Helmholtz equation). On the other hand, in the previous paper [2] the author gave a new formulation of the radiation condition applicable to general hyperbolic systems of Maxwell type, and used it to develop the spectral and scattering theory for the systems in an exterior domain. The acoustic and Maxwell's equations are typical examples of systems of Maxwell type, which suggests that the radiation condition defined in [2] implies both the Sommerfeld and the Silver-Müller radiation conditions. The subject of this note is to verify this by proving that our definition limited to the acoustic (resp. Maxwell's) equation is equivarent to Sommerfeld's (Silver-Müller's) one.

1. A radiation condition for reduced systems of Maxwell type. Let us consider symmetric systems of the form

$$Au=\sum_{j=1}^{n} A_j \frac{\partial u}{\partial x_j}=\lambda u \tag{1}$$

in an exterior domain G of $\boldsymbol{R}^n (n\geq 2)$. Here λ is an arbitrary non-zero complex number, $u=u(x)$ is a $\boldsymbol{C}^m$-valued function of $x=(x_1, x_2, \cdots, x_n) \in G$, and A_j are $m\times m$ Hermitian symmetric matrices with the property

The matrix $A(\xi)=\sum_{j=1}^{n} A_j\xi_j (\xi \in R^n-\{0\})$ *is isotropic, that is,*

$$\det [A(\xi)-\lambda I_m]=\prod_{\nu=1}^{k} (\tau_\nu |\xi|-\lambda)^{m_\nu}, \quad \sum_{\nu=1}^{k} m_\nu=m, \tag{2}$$

where I_m *is the identity in* $\boldsymbol{C}^m$, τ_ν *and* m_ν $(\nu=1, 2, \cdots, k)$ *are constants, and* $|\xi|=(\xi_1^2+\xi_2^2+\cdots+\xi_n^2)^{1/2}$. We say that the operator A is of Maxwell type if the matrix $A(\xi)$ is isotropic (cf., Wilcox [6]).

We lavel $\{\tau_\nu\}$ in decreasing order:

$$\tau_1>\tau_2>\cdots>\tau_k. \tag{3}$$