154. On Strongly Normal Spaces

By Yûkiti KATUTA
Ehime University

(Comm. by Kinjirô KUNUGI, M. J. A., Oct. 13, 1969)

A strongly normal space is a countably paracompact, collectionwise normal space (M. Katětov [2]). M. Katětov [2] and V. Šedivá [5] proved independently the following:

Theorem 1. A (Katětov, Šedivá). *A normal space X is a strongly normal space if and only if for every locally finite collection $\{F_\lambda \mid \lambda \in \Lambda\}$ of subsets of X there exists a locally finite collection $\{G_\lambda \mid \lambda \in \Lambda\}$ of open subsets of X such that $F_\lambda \subset G_\lambda$ for each $\lambda \in \Lambda$.*

We have, however, no informations on other characterizations of strongly normal spaces. The purpose of this paper is to obtain some characterizations of strongly normal spaces in terms of "coverings" (Theorem 2. A). Furthermore, we shall also obtain similar characterizations of collectionwise normal spaces (Theorem 2. B).

An open covering of a topological space is called an *A-covering* if it has a locally finite (not necessarily open) refinement. Every countable open covering is an A-covering. Indeed, for a countable open covering $\mathfrak{U}=\{U_n \mid n=1, 2, \cdots\}$ the collection $\{V_n \mid n=1, 2, \cdots\}$ is a locally finite refinement of $\mathfrak{U}$, where $V_1=U_1$ and $V_n=U_n-\bigcup_{i=1}^{n-1} U_i$ for $n=2, 3, \cdots$.

A collection of subsets of a topological space is called *bounded locally finite*, if there is a positive integer n such that every point of the space has a neighborhood which intersects only at most n elements of the collection. The following theorem is due to [2, Proposition 3.1].

Theorem 1. B (Katětov). *A normal space X is a collectionwise normal space if and only if for every bounded locally finite collection $\{F_\lambda \mid \lambda \in \Lambda\}$ of subsets of X there exists a locally finite collection $\{G_\lambda \mid \lambda \in \Lambda\}$ of open subsets of X such that $F_\lambda \subset G_\lambda$ for each $\lambda \in \Lambda$.*

An open covering of a topological space is called a *B-covering* if it has a bounded locally finite refinement. Of course, every B-covering is an A-covering and every finite open covering is a B-covering.

Theorem 2. A. *For a topological space X the following conditions are equivalent:*

(a) *X is a strongly normal space.*

(b) *X is a normal space[1] and for every locally finite covering*

1) In our terminology a normal space need not be a Hausdorff space.