153. On Mixed Problems for First Order Hyperbolic Systems with Constant Coefficients

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1. Introduction. Mixed problems for linear hyperbolic equations with constant coefficients in a quarter space has been treated by S. Agmon [1], R. Hersh [2] and L. Sarason [6].

In this note, we consider the mixed problem for first order hyperbolic systems with the principal part

$$(1.1)\qquad \begin{cases} L[u]\equiv \dfrac{\partial}{\partial t}u+A\dfrac{\partial}{\partial x}u+\sum_{j=1}^{n}B_j\dfrac{\partial}{\partial y_j}u=f(t\,;x,y)\\ u(0\,;x,y)=0\\ Pu(t\,;0,y)=0\end{cases}$$

in the quarter space $\{(t\,;x,y)\,;\,t>0,\,x>0,\,y\in R^n\}$, where u is a N-vector, A, $B_j(j=1,2,\cdots,n)$ $N\times N$-constant matrices and P $m\times N$-constant matrix of rank m. A is supposed to be non-singular.

Our argument is based on Wiener-Hopf's method. After Laplace transformation in t and Fourier transformation in y, the problem (1.1) is translated into the following equation

$$(1.2)\qquad \begin{cases}\left(A\dfrac{d}{dx}+\tau I+i\sum_{j=1}^{n}\eta_jB_j\right)\hat{u}(\tau\,;x,\eta)=\hat{f}(\tau\,;x,\eta)\\ P\hat{u}(\tau\,;0,\eta)=0,\end{cases}$$

where $\hat{u}(\tau\,;x,\eta)$ denotes the Fourier-Laplace image of $u(t\,;x,y)$. Using a compensating function $\hat{g}(\tau\,;x,\eta)$ which shall be constructed later and setting $u=v+w$, we decompose the problem (1.2) to two problems

$$(1.3)\qquad \left(A\frac{d}{dx}+\tau I+i\sum_{j=1}^{n}\eta_jB_j\right)\hat{v}(\tau\,;x,\eta)=\hat{f}(\tau\,;x,\eta)+\hat{g}(\tau\,;x,\eta)$$

in $x\in R^1$ and

$$(1.4)\qquad \begin{cases}\left(\dfrac{d}{dx}+M(\tau,\eta)\right)\hat{w}(\tau\,;x,\eta)=0\\ P\hat{w}(\tau\,;0,\eta)=-P\hat{v}(\tau\,;0,\eta)\end{cases}$$

where $M(\tau,\eta)=A^{-1}\left(\tau I+i\sum_{j=1}^{n}\eta_jB_j\right)$. Thus we are to treat the problems (1.3) and (1.4).

2. Assumptions and result. *Condition* I. The operator L is hyperbolic in the following sense: 1) the matrix $\xi A+\eta B$ $\left(\eta B\text{ stands for }\sum_{j=1}^{n}\eta_jB_j\right)$ has only real eigenvalues for any real (ξ,η), 2) the matrix