150. 5-dimensional Orientable Submanifolds of R^7. II

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1. Introduction. In our previous paper [4], we showed that, using the vector cross product induced by Cayley numbers, any 5-dimensional orientable submanifold M of R^7 admits an almost contact structure.

In this paper, denoting this almost contact structure by (ϕ, ξ, η), we shall study the torsion of ϕ. First, we shall prove that if M is totally geodesic then the torsion of ϕ vanishes identically (Theorem 1). Secondly, we consider the converse problem. Unfortunately, this is not true in general. But we shall prove that if M is totally umbilical, then the vanishing of the torsion of ϕ implies that M is totally geodesic (Theorem 2).

2. Basic informations.

(a) Almost contact manifolds.

Let M be a $(2n+1)$-dimensional C^∞ manifold with an almost contact structure (ϕ, ξ, η). Then we have, by definition,

$$\eta(\xi)=1, \tag{1}$$

$$\phi(\xi)=0, \qquad \eta\circ\phi=0, \tag{2}$$

$$\phi^2=-I+\eta(\cdot)\xi, \tag{3}$$

where I is the identity transformation field.

By above relations, it can be easily shown that the rank of ϕ is $2n$.

We denote the associated Riemannian metric of (ϕ, ξ, η) by $\langle\ ,\ \rangle$. Then it satisfies

$$\eta=\langle\xi, \cdot\rangle, \tag{4}$$

(5) $\langle\phi X, \phi Y\rangle=\langle X, Y\rangle-\eta(X)\eta(Y)$, for any vector fields X, Y on M.

The tensor $N(X, Y)$ defined by

$$N(X, Y)=[X, Y]+\phi[\phi X, Y]+\phi[X, \phi Y]-[\phi X, \phi Y] \\ -\{X\cdot\eta(Y)-Y\cdot\eta(X)\}\xi \tag{6}$$

is called the *torsion* of ϕ and M is called *normal* if N vanishes identically.

(b) The vector cross product on R^7.

The vector cross product on R^7 is a linear map $P: V(R^7)\times V(R^7) \to V(R_7)$ (writing here $P(\bar{X}, \bar{Y})=\bar{X}\otimes\bar{Y}$) satisfing the following conditions:

$$\bar{X}\otimes\bar{Y}=-\bar{Y}\otimes\bar{X}, \tag{7}$$

$$\langle\bar{X}\otimes\bar{Y}, \bar{Z}\rangle=\langle\bar{X}, \bar{Y}\otimes\bar{Z}\rangle, \tag{8}$$