149. A Remark on a Semilinear Degenerate Diffusion System

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§1. **Introduction.** This remark is concerned with the following mixed problem in $R^T=\{0<t\leqslant T, 0<x\}$,

$$(1)\qquad \frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+f(v)u+g(v), \qquad \frac{\partial v}{\partial t}=u,$$

with the initial boundary conditions,

$$(2)\qquad \begin{aligned} &u(x, 0)=u_0(x), \qquad v(x, 0)=v_0(x) \qquad \text{for } 0\leqslant x\\ &v(0, t)=\psi(t) \qquad \text{for } 0\leqslant t\leqslant T. \end{aligned}$$

First, let us note the theorem proved by R. Arima and Y. Hasegawa [1] with respect to the problem (1) and (2), which is given as follow:

Theorem 1. *Suppose,*

$$(3)\qquad \begin{aligned} &f(v),\ g(v)\in C^1,\\ &-K_1(v^2+1)\leqslant f(v)\leqslant L,\\ &|g(v)|\leqslant K_2(v^2+|v|) \quad \text{and} \quad G(v)\equiv\int_0^v g(z)dz\leqslant K_3v^2,\\ &u_0(x),\quad v_0(x)\in \mathcal{B}^2_+\cap \mathcal{D}^2_{L^2+} \qquad \text{for } 0\leqslant x,\\ &\psi(t)\in C^2 \qquad \text{for } 0\leqslant t\leqslant T,\\ &u_0(0)=\varphi'(0),\quad v_0(0)=\varphi(0),\\ &\psi''(0)=u_0''(0)+f(\psi(0))\psi'(0)+g(\psi(0)). \end{aligned}$$

Then there exists a unique solution $\{u(x, t), v(x, t)\}$ *in* R^T *such that* $\{u(x, t), v(x, t)\}\in \mathcal{E}^0_t(\mathcal{B}^2_+\cap\mathcal{D}^2_{L^2+})$, *where* $L, K_1, K_2,$ *and* K_3 *are positive constants.*

In this note we prove the existence and the uniqueness theorem of the following more general system than (1) by using a suitable difference scheme,

$$(4)\qquad \begin{aligned} &\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+f(v)u+g(v)\\ &\frac{\partial v}{\partial t}=a(u)v+b(u) \end{aligned}$$

and drive the different conditions from (3) in the case of $a(u)\equiv 0$ and $b(u)\equiv u$.

Here we consider the mixed problem in R^T for (4) with the initial boundary conditions,

$$(5)\qquad \begin{aligned} &u(x, 0)=u_0(x), \qquad v(x, 0)=v_0(x) \qquad \text{for } 0\leqslant x\\ &u(0, t)=\varphi(t), \qquad v(0, t)=\psi(t) \qquad \text{for } 0\leqslant t\leqslant T, \end{aligned}$$

and also the compatibility conditions,