148. *Korteweg-deVries Equation.* IV

Simplest Generalization

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1. Introduction. In the preceding note [1], [2] we have established the global existence theorem for the Cauchy problem for the KdV equation

$$(1)\qquad \begin{cases} D_t u + uDu + D^3u = 0 & (x,t) \in R^1 \times [0,\infty) \\ u(x,0) = f(x) & x \in R^1 \quad \left(D_t = \dfrac{\partial}{\partial t},\ D = \dfrac{\partial}{\partial x}\right). \end{cases}$$

Here we consider its simplest generalization

$$(2)\qquad \begin{cases} D_t v + v^2 Dv + D^3 v = 0 & (x,t) \in R^1 \times [0,\infty) \\ v(x,0) = f(x) & x \in R^1. \end{cases}$$

These Cauchy problems are closely related to the study of anharmonic lattices [3]. Recently Miura [4] has discovered a remarkable nonlinear transformation

$$(3)\qquad \sqrt{-6}Dv + v^2 = u$$

which connects (1) and (2) in the following manner

$$(4)\qquad D_t u + uDu + D^3 u = (2v + \sqrt{-6}D)(D_t v + v^2 Dv + D^3 v).$$

For any smooth solution $v(\in \mathcal{E}_t^\infty(\mathcal{E}_{L^2}^\infty))$ of (2) we can associate uniquely the solution $u(\in \mathcal{E}_t^\infty(\mathcal{E}_{L^2}^\infty))$ of (1) through the transformation (3). But converse is not true. When we want to solve the equation (3) with respect to v for given u we have no uniqueness. First we show the example which violates the uniqueness of the solutions of the equation (3). Let $\varphi(x) \in \mathcal{E}_{L^2}^\infty$ be such that $\varphi(x) > 0$ for $\forall x \in R^1$, $\varphi(x) = \dfrac{1}{|x|}$ for $|x| > R$ for some $R > 0$. We define v, w, u as follows

$$v = \frac{1}{2}\varphi - \frac{\sqrt{-6}}{2}\frac{D\varphi}{\varphi}, \qquad w = v - \varphi, \qquad u = \sqrt{-6}Dv + v^2.$$

Then v and w are distinct each other and satisfies the same equation (3). That is the violence of the uniqueness of the equation (3). Therefore the global existence theorem for the Cauchy problem (1) is insufficient for the global existence theorem for the Cauchy problem (2). We establish here the global existence theorem for the Cauchy problem (2) in a slightly general situation. Detailed proof will be published elsewhere.