147. Korteweg-deVries Equation. III

Global Existence of Asymptotically Periodic Solutions

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1. Introduction. In the preceding note [1] we show the global existence of the smooth solutions of the Cauchy problem for the KdV equation. That is for any initial data $f(x) \in \mathcal{E}^\infty_{L^2}(R^1) = \mathcal{E}^\infty_{L^2}$ and for any inhomogeneous term $g(x, t) \in \mathcal{E}^\infty_t(\mathcal{E}^\infty_{L^2})$ the Cauchy problem for the KdV equation

$$\begin{cases} D_t u + uDu + D^3 u + g(x, t) = 0 & (x, t) \in R^1 \times [0, \infty) \\ u(x, 0) = f(x) \quad x \in R^1 & \left(D_t = \dfrac{\partial}{\partial t}, \quad D = \dfrac{\partial}{\partial x}\right) \end{cases}$$

has uniquely the global solution $u(x, t) \in \mathcal{E}^\infty_t(\mathcal{E}^\infty_{L^2})$ $(0 \leq t < \infty)$. Moreover we may replace the functional space $\mathcal{E}^\infty_{L^2}$ by the functional space $\mathcal{P}^\infty_l$ (see Definition 1).

In this note we extend slightly the preceding results [1] and show the global existence of the asymptotically periodic solutions of the Cauchy problem for the KdV equation.

Detailed proof will be published elsewhere.

2. Global existence theorems.

Definition 1. $f(x)$ belongs to the functional space $\mathcal{P}^k_l$ if and only if $f(x)$ is a periodic function with period l, and belongs to $\mathcal{E}^k_{L^2\mathrm{loc}}(R^1)$ (k is a non negative integer or ∞).

Definition 2. $f(x)$ is called asymptotically periodic if and only if $f(x)$ belongs to the functional space $Q^k_l = \mathcal{P}^k_l + \mathcal{E}^k_{L^2}$ for some k and l. Here $+$ signe represents the direct sum of the two Hilbert spaces $\mathcal{P}^k_l$ and $\mathcal{E}^k_{L^2}$.

Consider the Cauchy problem for the KdV equation (with dissipative lower order terms)

$$(1) \quad \begin{cases} D_t u + uDu + D^3 u - \mu D^2 u + a(x, t)Du + b(x, t)u + g(x, t) = 0 \\ \qquad\qquad\qquad\qquad (x, t) \in R^1 \times [0, \infty) \\ u(x, 0) = f(x) \qquad\qquad x \in R^1 \end{cases}$$

Assumption 1. $\mu \geq 0$. $\quad a(x, t), b(x, t) \in \mathcal{E}^\infty_t(\mathcal{P}^\infty_l)$

Main theorem. We assume Assumption 1. For any initial data $f(x) = f_0(x) + f_1(x) \in Q^\infty_l$ and for any inhomogeneous term $g(x, t) = g_0(x, t) + g_1(x, t) \in \mathcal{E}^\infty_t(Q^\infty_l)$ the Cauchy problem for the KdV equation (1) has uniquely the global solution $u(x, t) \in \mathcal{E}^\infty_t(Q^\infty_l)$ $(0 \leq t < \infty)$. Moreover