146. *Free and Semi-free Differentiable Actions on Homotopy Spheres*

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1. Introduction. The theorem of Browder-Novikov enables us to construct free differentiable actions of S^1 and S^3 on homotopy spheres (see Hsiang [8]). As is shown in § 2 of this paper, every free differentiable action is obtained in such a way. Hence if we know J-groups of complex projective spaces CP^n and quaternionic projective spaces QP^n, we can classify free differentiable actions of S^1 and S^3 on homotopy spheres. In [12], Prof. S. Sasao has determined J-groups of spaces which are like projective planes. Consequently we can determine the homotopy 11-spheres admitting free differentiable S^3-actions. Let Σ_M^{11} be the generator of Θ_{11} due to Milnor. Then we shall have

Theorem 1. *Every homotopy sphere Σ which is diffeomorphic to $32k\,\Sigma_M^{11}$ for some $k\equiv 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 10, \pm 11, \pm 12, \pm 14, \pm 15 \pmod{31}$ admits infinitely many topologically distinct S^3-actions and the remains of homotopy 11-spheres do not admit any free differentiable S^3-actions.*

Let Θ_n be the group of homotopy n-spheres and $\Theta_n(\partial\pi)$ be the subgroup consisting of those homotopy spheres which bound parallelizable manifolds. Let $\beta : \Theta_n \to \Theta_n(\partial\pi)$ be the splitting due to Brumfiel [5] and let Σ_M^{15} be the generator of $\Theta_{15}(\partial\pi)$ due to Milnor. Then we shall have

Theorem 2. *There exist at least 35 homotopy 15-spheres $\{\Sigma_k\}$ all of which admit infinitely many topologically distinct S^3-actions such that $\beta(\Sigma_k)=2^6\cdot k\Sigma_M^{15}$ where $k\equiv 0, \pm 6, \pm 8, \pm 13, \pm 14, \pm 15, \pm 17, \pm 23, \pm 26, \pm 34, \pm 35, \pm 45, \pm 48, \pm 50, \pm 51, \pm 53, \pm 55, \pm 57$ (mod. 127).*

On the other hand we shall have

Theorem 3. *A homotopy 15-sphere Σ admits no free differentiable S^3-actions if $k\not\equiv 4 \pmod 4$ where k is an integer defined by $\beta(\Sigma)=k\Sigma_M^{15}$.*

As for free S^1-actions on homotopy 15-spheres, we shall have

Theorem 4. *There exist at least 70 homotopy 15-spheres $\{\Sigma_i^{15}\}$ all of which admit infinitely many topologically distinct S^1-actions.*

An action (M^m, φ, G) is called semi-free if it is free off of the fixed

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