# 145. *On the Class Number of an Absolutely Cyclic Number Field of Prime Degree*

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Let $K$ be a cyclic extension of odd prime degree $p$ over $\boldsymbol{Q}$, and suppose that 2 is a primitive root mod $p$. $p$ may be, for example, 3, 5, 11, 13, 19 or 29. We shall prove that the class number $h$ of $K$ is even, if and only if a cyclotomic unit $\eta$ of $K$ is either totally positive or totally negative, i.e. $|\eta|$ is totally positive. We shall also show that $|\eta|$ is not totally positive, if the discriminant of $K$ is a power of prime. Hence, in such a case, we can conclude that the class number $h$ of $K$ is odd.

## §1. On cyclotomic units.

In order to prove our results, we first recollect some properties of cyclotomic units, which are described in [3] with thorough proofs.

Let $K$ be a cyclic extension of odd prime degree $p$ over $\boldsymbol{Q}$. Then, it is well known that $K$ is cyclotomic, that is, $K$ is contained in $\boldsymbol{Q}_m=\boldsymbol{Q}(\zeta_m)$ for some $m$. Here, and in what follows, $\zeta_m$ denotes

$$\cos\frac{2\pi}{m}+i\sin\frac{2\pi}{m}.$$

Let $f$ be the greatest common divisor of $m$'s such that $\boldsymbol{Q}_m\supset K$. Then, $K$ is contained in $\boldsymbol{Q}_f$. Note that a prime number is ramified in $K$, if and only if it divides $f$. For any integer $a$ which is prime to $f$, we define the element $i(a)$ of the Galois group $G(\boldsymbol{Q}_f/\boldsymbol{Q})$ by

$$\zeta_f^{i(a)}=\zeta_f^a.$$

Then the map

$$a\mapsto i(a)$$

induces an isomorphism of the multiplicative group $\boldsymbol{Z}_f^\times$ of reduced residue classes mod $f$ onto $G(\boldsymbol{Q}_f/\boldsymbol{Q})$. We will use the same notation $i(a)$ for this isomorphism. In general, we will write $a$ for the class of $a$ mod $f$. Denote by $i_K(a)$ the element of $G(K/\boldsymbol{Q})$ which is induced by $i(a)$. Then, the map

$$a\mapsto i_K(a)$$

induces a homomorphism of $\boldsymbol{Z}_f^\times$ onto $G(K/\boldsymbol{Q})$. We denote by $H$ the kernel of this homomorphism. Since $K$ is real, all elements of $K$ are invariant by $\zeta_f\mapsto\zeta_f^{-1}$. Hence, $-1$ is contained in $H$. We take a subset $A$ of $H$ such that $A\cup\{-a;a\in A\}=H$, and $A\cap\{-a;a\in A\}=\emptyset$. Let $s$