173. On the Classical Stability Theorem of Poincaré-Lyapunov with a Random Parameter

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1. The objective of this paper is concerned with the generalization of the classical stability theorem of Poincaré-Lyapunov, [1].

The Poincaré-Lyapunov theorem with a random parameter can be written as follows:

$$\dot{x}(t;\omega)=A(\omega)x(t;\omega)+f(t,x(t;\omega)),\quad t\geq 0 \tag{1.0}$$

where

(i) $\omega\in\Omega$, Ω being the supporting set of the probability measure space (Ω, A, μ);

(ii) $x(t;\omega)$ is the unknown nxl random vector;

(iii) $A(\omega)$ is mxn matrix whose elements are measurable functions;

(iv) $f(t,x)$ is for $t\in R_+$ and $x\in R$ an nxl vector valued function.

The above random differential system can be easily reduced into the following stochastic equation

$$x(t;\omega)=e^{A(\omega)t}x_0(\omega)+\int_0^t e^{A(\omega)(t-\tau)}f(\tau, x(\tau;\omega))d\tau. \tag{1.1}$$

Remark. The term $e^{A(\omega)t}x_0(\omega)$ is referred to as the free stochastic term or free random variable, $e^{A(\omega)(t-\tau)}$ the stochastic kernel and $x(0,\omega)=x_0(\omega)$.

The particular aim of this paper is the existence, uniqueness and asymptotic behavior of a random solution of the stochastic integral equation (1.1). In accomplishing this objective we utilized certain aspects and methods of "admissibility theory" which can be found in [2].

2. We shall consider that the random solution $x(t;\omega)$ and the stochastic free term $e^{A(\omega)t}x_0(\omega)$ are functions of the real argument t with values in the space $L_2(\Omega, A, \mu)$. The function $f(t, x(t;\omega))$, under convenient conditions, will also be a function of t with values in $L_2(\Omega, A, \mu)$. The value of the stochastic kernel, $e^{A(\omega)(t-\tau)}$, $0\leq\tau\leq t$, shall be an essentially bounded function with respect to μ for every t and τ, such that $0\leq\tau\leq t<\infty$. The values of this term for fixed t and τ, will be in $L_\infty(\Omega, A, \mu)$ so that the product of $e^{A(\omega)t}x_o(\omega)$ and $e^{A(\omega)(t-\tau)}$ will always be in $L_2(\Omega, A, \mu)$.

The norm of the stochastic kernel of the random integral equation