171. *On the Nörlund Summability of Fourier Series*

By Masako IZUMI and Shin-ichi IZUMI
Department of Mathematics, The Australian National University, Canberra, Australia

(Comm. by Zyoiti SUETUNA, M. J. A., Nov. 12, 1969)

1. Introduction and Theorems.

1.1. Definitions. Let $\sum a_n$ be a given series and s_n be its nth partial sum. Let (p_n) be a sequence of real numbers such that $p_0=0$, $P_n=p_0+p_1+\cdots+p_n\neq 0$ for all n and $|P_n|\to\infty$ as $n\to\infty$. If the sequence

$$t_n=\frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}s_k \qquad (n=1, 2, \cdots) \tag{1}$$

tends to a limit s as $n\to\infty$, then the series $\sum a_n$ is said to be (N, p_n) summable to s. This method of summation is regular if and only if

$$\sum_{k=0}^{n} |p_k|\leqq A|P_n| \qquad \text{for all } n\geqq 1 \text{ and } p_n/P_n\to 0 \text{ as } n\to\infty. \tag{2}$$

Let f be an integrable function with period 2π and its Fourier series be

$$f(t)\sim\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)=\sum_{n=0}^{\infty}A_n(x). \tag{3}$$

We write $\varphi(t)=\varphi_x(t)=f(x+t)+f(x-t)-2f(x)$.

1.2. E. Hille and J. D. Tamarkin [1] have applied the (N, p_n) summation to Fourier series. Extending one of their theorems, O. P. Vershney [2] has proved the following

Theorem I. *Suppose that the sequence (p_n) of real numbers satisfies the conditions*:

$$n|p_n|\leqq A|P_n|\log(n+1) \qquad \textit{for } n\geqq 1, \tag{4}$$

$$\sum_{k=1}^{n}\frac{k|\Delta p_k|}{\log(k+1)}\leqq A|P_n| \qquad \textit{for all } n\geqq 1 \tag{5}$$

and

$$\sum_{k=1}^{n}\frac{|P_k|}{k\log(k+1)}\leqq A|P_n| \qquad \textit{for all } n\geqq 1. \tag{6}$$

If

$$\Phi(t)=\int_0^t|\varphi(u)|\,du=o\left(t/\log\frac{1}{t}\right) \qquad \textit{as } t\to 0, \tag{7}$$

then the Fourier series of f is (N, p_n) summable to $f(x)$.

On the other hand O. P. Vershney [3] proved the

Theorem II. *Let (p_n) be a positive non-increasing sequence. Then the Fourier series of f satisfying the condition*

$$\varphi(t)=o\left(1/\log\frac{1}{t}\right) \qquad \textit{as } t\to 0,$$