201. *A Characterization of Pseudo-open Images of M-Spaces*

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P. S. Alexandroff [1] asked which spaces can be represented as images of "nice" spaces under "nice" continuous maps. Until recently, the "nice" spaces have generally consisted of metric spaces. In 1968, J. Nagata rephrased the question, "which spaces can be represented as images of M-spaces under 'nice' continuous maps." He has since characterized quotient and bi-quotient images of M-spaces in [6], and later he answered the question for open images of M-spaces [4].

Consider the following

Definition 1. A map $f: X \rightarrow Y$ is said to be pseudo-open iff, given any $y \in Y$ and U open about $f^{-1}(y)$ in X, then $y \in \operatorname{int} f(U)$.

E. Michael and Nagata, working independently, suggested essentially the following characterization:

Theorem 1. *A space Y is the pseudo-open image of an M-space X iff Y has the following property*:

$y \in \mathrm{Cl}\, B$ *for* $B \subseteq Y$ *implies that there is a family* $\{U_1, U_2, \cdots\}$ *of subsets of Y such that*

(1) $y \in U_i$ *for all* i;

(2) $y \in \mathrm{Cl}(U_i \cap B)$ *for all* i,

(3) *If* $x_i \in U_i$ *for all* i, *then the sequence* $\{x_i\}$ *has a cluster point* x' *in* $\bigcap_{i=1}^{\infty} U_i$.

Definition 2. A space satisfying the above property will be said to have property-(P). The Nagata-Michael conjecture is correct, as this paper will demonstrate. First, recall some information from Nagata [6].

Definition 3. A sequence $A_1 \supseteq A_2 \supseteq \cdots$ of subsets of a space X is called a q-sequence if any point sequence $\{x_i : i=1, 2, \cdots\}$ satisfying $x_i \in A_i$ has a cluster point in $\bigcap A_i$. (This is (3) of property-(P).)

Definition 4. A sequence $U_1, U_2, \cdots$ of open neighborhoods of a point x in X is called a q-sequence of neighborhoods if $U_1 \supseteq \bar{U}_2 \supseteq U_2 \cdots$ and if any point sequence $\{x_i : i=1, 2, \cdots\}$ satisfying $x_i \in U_i$ has a cluster point.

Lemma 1. *Let f be a continuous map from a space X onto a space Y. If $\{A_i : i=1, 2, \cdots\}$ is a q-sequence in X, then $\{f(A_i) : i, 2, \cdots\}$*