198. *Two Spaces whose Product has Closed Projection Maps*

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This note will give several equivalent properties with that of two spaces in the title and some properties of the spaces. This is also a preparation for the forthcoming paper [2].

Throughout this note, spaces are Hausdorff. We use notations in [1].

Definition 1 (cf. [4, p. 365]). A set A in $X\times Y$ is called to be *upper semi-continuous at* $a\in X$ if for any open set G in Y containing $A[a]$ there is $U\in\mathfrak{N}_a$ with $\bigcup_{x\in U}A[x]\subset G$. A is called *upper semi-continuous at* X if A is upper semi-continuous at every point of X.

It is easily seen that A is upper semi-continuous at X if and only if the set $\{x\in X\,;\,A[x]\subset G\}$ is open in X for every open G of Y.

Definition 2. Let a be a point of X. A space Y with the following property is called to be *upper compact at* a. Let Z be any subset of X with $a\in\bar{Z}$, and let $\{A_x\,;\,x\in Z\}$ be any family of non-empty subsets of Y, then $\limsup_a A_x\neq\emptyset$. Y is called *upper compact at* X when Y is upper compact at every point of X.

In this definition we can replace $\bar{Z}$ by $\bar{Z}-Z$.

The following is seen easily.

Proposition 1. *A closed subset of a space which is upper compact at* $a\in X$ *is upper compact at* a.

Proposition 2. *In order that* Y *is upper compact at* $a\in X$, *it is necessary and sufficient that for any subset* Z *of* X *with* $a\in\bar{Z}$, *and for any family* $\{B_U\,;\,U\in\mathfrak{N}_a\}$ *of subsets of* Y *such that*

$$\bigcap_{U\ni x}\overline{B_U}\neq\emptyset$$

for every point $x\in Z$, *it holds* $\bigcap_{U\in\mathfrak{N}_a}\overline{B_U}\neq\emptyset$.

Proof. *Necessity.* Put

$$A_x=\bigcap_{U\ni x}\overline{B_U}$$

for $x\in Z$, then A_x is not empty, so

$$\emptyset\neq\bigcap_{U\in\mathfrak{N}_a}\overline{\bigcup_{x\in U}A_x}\subset\bigcap_{U\in\mathfrak{N}_a}\overline{B_U}.$$

Sufficiency. Let $\{A_x\,;\,x\in Z\subset X\}$, $a\in\bar{Z}$, be an arbitrary family of non-empty subsets of X. Put