196. *Z-mappings and C*-embeddings*

By Takesi ISIWATA

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Recently, Comfort and Negrepontis investigated the interesting properties of proper C*-pair [1]. In this paper, we shall give in § 1 a necessary and sufficient condition that $X\times Y$ is C^*-embedded in $X\times \beta Y$ and give in § 2 partial answers to the problems with respect to the product spaces raised by Morita.

Throughout this paper, we assume that our spaces are completely regular T_1-spaces and mappings are continuous. We will use the same notations as in [3]; for instance, the symbol βX denotes the Stone Čech compactification of a given space X. We denote by λ the projection: $X\times Y\to X$ and put $W=X\times \beta Y$.

§ 1. Relations between *Z*-mappings and *C-embeddings.**

We call a mapping φ from X onto Y a *Z-mapping* if φE is closed in Y for every zero set E of X. A closed mapping is always a Z-mapping ([5], 1.1).

1.1. Theorem. *$X\times Y$ is C^*-embedded in $X\times \beta Y$ if and only if the projection $\lambda: X\times Y\to X$ is a Z-mapping.*

Proof. *Necessity.* Let F be a zero set of $X\times Y$; that is, there is a function $f\in C^*(X\times Y)$ such that $F=\{(x,y); f(x,y)=0\}$ and $0\leq f\leq 1$ on $X\times Y$. Now suppose that there exists a point $x_0\in \mathrm{cl}\,\lambda F-\lambda F$. Since βY is compact, the projection $\pi: W=X\times \beta Y\to X$ is closed. $\mathrm{Cl}_W F$ being closed, $\pi(\mathrm{cl}_W F)$ contains x_0. On the other hand, $x_0\notin F$ implies that f is positive on $\{x_0\}\times Y$. We shall consider the function g defined in the following way:

$$g(x,y)=(f\,|\,(\{x_0\}\times Y))(x_0,y) \quad \text{for} \quad (x,y)\in X\times Y.$$

It is easy to see that g is continuous and $0\leq g\leq 1$. Define

$$h(x,y)=(f(x,y)/g(x,y))\wedge 1.$$

The function h is continuous and $F=Z(h)$ and $h=1$ on $\{x_0\}\times Y$. We denote by k the continuous extension of h over $X\times \beta Y$. Obviously $k=1$ on $\{x_0\}\times \beta Y$. This shows that $\mathrm{cl}_W F\cap\{x_0\}\times \beta Y=\emptyset$ which is impossible.

Sufficiency. In Theorem 3.1 in [1], it is proved that if λ is closed, then $X\times Y$ is C^*-embedded in $X\times \beta Y$. In its proof, it is easy to check that "closedness of λ" is replaced by "λ being a Z-mapping".

Remark. $X\times Y$ is not necessarily C^*-embedded in $\beta X\times \beta Y$ even if both projections: $X\times Y\to X$ and $X\times Y\to Y$ are Z-mappings (for instance, both spaces X and Y are discrete [1], [4]).