195. On a Set Theory Suggested by Dedecker and Ehresmann. II

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The preceding theorem shows how the general notions, referring to classes, are related to the corresponding ones, *restricted* to sets. If we wish to extend the above relations to the other concepts of set theory like, for instance, the notions of ordered pair, function, ordinal number, and cardinal number, it is clear that several modifications must be made in the general definitions (borrowed from [13]) or in the definitions of the *restricted* notions (adapted from [9], appendix). However as we have defined V-union ($\cup_V$), V-intersection ($\cap_V$) and V-complement ($\sim_V$), we may define V-ordered pair, V-function, V-choice function, etc., following [9].

The behaviour of sets is regulated by the following postulates:

Postulate of subsets.

(P5) $$x \in V \supset \exists y(y \in V \ \&\ \forall z(z \subset x \supset z \in y)).$$

Postulate of union.

(P6) $$x \in V \ \&\ y \in V \supset x \cup_V y \in V.$$

Postulate of substitution.

(P7) $$x \text{ is a } V\text{-function } \&\ \mathrm{dom}_V x \in V \supset \mathrm{range}_V x \in V.$$

Postulate of amalgamation.

(P8) $$x \in V \supset \cup_V x \in V.$$

Postulate of regularity.

(P9) $$x \in V \ \&\ x \neq \emptyset \supset \exists z(z \in x \ \&\ x \cap_V z = \emptyset).$$

Postulate of infinity.

(P10) $$\exists y(y \in V \ \&\ \emptyset \in y \ \&\ \forall z(x \in y \supset x \cup_V \{x\}_V \in V)).$$

Theorem 9. $\vdash \exists x(x \in V)$.

Corollary. $\vdash V \neq \emptyset$.

Postulate of choice.

(P11) $$\exists x(x \text{ is a } V\text{-choice function } \&\ \mathrm{dom}_V x = \{y : y \neq \emptyset\}_V).$$

From the above postulates, we may deduce that all the theorems of the Kelley-Morse system, conveniently translated, are true for the elementary classes of $\boldsymbol{D}$. Thus, $\boldsymbol{D}$ is strictly stronger than the Kelley-Morse set theory.