

**194. On a Set Theory Suggested by Dedecker  
and Ehresmann. I<sup>\*</sup>**

By Newton C. A. da COSTA

Institute of Mathematics, University of Campinas,  
Campinas, São Paulo, Brazil

and

Institute of Mathematics, Universidad Nacional del Sur,  
Bahia Blanca, Argentina

(Comm. by Kinjirô KUNUGI, M. J. A., Dec. 12, 1969)

**1. Introduction.** As it is known, the common systems of set theory do not provide an adequate foundation for the theory of categories (see, for example [2] and the references given there). Various solutions have been proposed to surmount this difficulty (cf. [2]-[3], [7], [8] and [14]). Using an idea of Ehresmann [6], Dedecker informally describes in [5] a set theory reputedly appropriate to serve as a basis for category theory. The object of the present paper is to formalize Dedecker's system (or, more precisely, to describe a formal system belonging to the type suggested by Dedecker and Ehresmann).

We were led to the formalization of dedecker's system, called here system **D**, studying questions of a very different nature (cf. [4]). In fact, **D** is the first of a hierarchy of set theories (similar to the hierarchy defined in [1]) which will be studied in the near future.

In systems of the Von Neumann-Bernays-Gödel type, like the Kelley-Morse set theory ([9], appendix), a distinction is made between sets and classes and one is able to operate on sets with the classical rules, but the same is in general not true of the operations with classes. In **D**, on the contrary, it is possible to *operate* on classes (proper or not) as one does with sets; for instance, the class of equivalence classes of a given class, corresponding to an equivalence relation always exists, and the unit class of any class is in all cases defined, such notions having the desired suitable properties.

In a few words, **D** is a combination of the Kelley-Morse set theory with the Quine-Rosser NF system ([12] and [13]).

(We presuppose that the reader has a good knowledge of [5], [9] and [13].)

---

<sup>\*</sup>) Partially supported by NATO Research Grant no. 224. The writer wishes to express his gratitude to Professor J. B. Rosser for commenting upon several specific points of the paper and for indicating various results incorporated in the text.