188. On Certain Mixed Problem for Hyperbolic Equations of Higher Order. III

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1. Introduction. Let $\boldsymbol{R}_+^n$ be the open half space $\{(x, y); x>0, y \in \boldsymbol{R}^{n-1}\}$. We consider the mixed problem $(P, B_j; j=1, \cdots, l)$, briefly (P, B_j), for hyperbolic equations of order m in $(0, T)\times \boldsymbol{R}_+^n$ $(0<T<\infty)$:

$$(P(D_t, D_x, D_y)u)(t, x, y)=f(t, x, y) \quad \text{in } (0, T)\times \boldsymbol{R}_+^n,$$
$$(B_j(D_t, D_x, D_y)u)(t, 0, y)=0 \ (j=1, \cdots, l) \quad \text{in } (0, T)\times \boldsymbol{R}^{n-1},$$
$$(D_t^k u)(0, x, y)=0 \ (k=0, 1, \cdots, m-1) \quad \text{in } \boldsymbol{R}_+^n,$$

where $D_t=\frac{\partial}{\partial t}$, $D_x=-i\frac{\partial}{\partial x}$, $D_y=\left(-i\frac{\partial}{\partial y_1}, \cdots, -i\frac{\partial}{\partial y_{n-1}}\right)$ and $i=\sqrt{-1}$.

The purpose of this paper is to determine the necessary and sufficient conditions for L^2-well-posedness in the following sense.

Definition. *The mixed problem (P, B_j) is L^2-well-posed if and only if there exist constants T and T' with $0<T'\leq T$ which satisfy the following condition*:

For every $f \in H^1((-\infty, T)\times \boldsymbol{R}_+^n)$ with $f=0$ $(t<0)$ the mixed problem (P, B_j) has a unique solution $u \in H^m((0, T')\times \boldsymbol{R}_+^n)$ so that

$$\sum_{k=0}^{m-1}\int_0^{T'} \|(D_t^k u)(t, \cdot, \cdot)\|_{m-k-1}^2 dt \leq C\int_0^T \|f(t, \cdot, \cdot)\|_0^2 dt,$$

where a constant C depends only on T.

In § 2 we give certain necessary and sufficient conditions for L^2-well-posedness (Theorem 1) and investigate zeros of the Lopatinskii's determinant under L^2-well-posedness (Theorem 2).

In T. Shirota and K. Asano [5] it has been shown by semi-group method that the mixed problem $(P, D_x^{2j-1}; j=1, \cdots, l)$ $(m=2l)$ is *well posed in the L^2-sense*[1] if $P(D)=P(D_t, D_x, D_y)$ does not contain the terms of odd order relative to D_x. As one of the applications of Theorems 1 and 2 we show that, in the case of constant coefficients, the above condition for $P(D)$ is necessary to be *well posed in the L^2-sense* for the above mixed problem. This assertion is found in Theorem 4 in § 3.

The details and other results will be published elsewhere.

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1) This term means that in our definition one changes the inequality into the energy inequality.