187. *Two Perturbation Theorems for Contraction Semigroups in a Hilbert Space*

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The purpose of this note is to prove two perturbation theorems for contraction semigroups in a Hilbert space. A linear operator T in a Hilbert space H is said to be *accretive* if $\mathrm{Re}(Tu, u) \geqq 0$ for all $u \in \mathrm{D}(T)$ ($\mathrm{D}(T)$ denotes the domain of T). If T satisfies the conditions that $(T+\xi)^{-1} \in \mathscr{B}(\mathrm{H})$ ($\mathscr{B}(\mathrm{H})$ is the set of all bounded operators from H to H) and $\|(T+\xi)^{-1}\| \leqq \xi^{-1}$ for $\xi > 0$, T is said to be *m-accretive*. Let T and A be operators in H such that

(1) $$\|Au\| \leqq a\|u\| + b\|Tu\|, \; u \in \mathrm{D}(T) \subset \mathrm{D}(A),$$

where a, b are nonnegative constants. Then we say that A is *relatively bounded with respect to* T or simply *T-bounded*. The condition (1) is equivalent to

(2) $$\|Au\|^2 = a'^2\|u\|^2 + b'^2\|Tu\|^2, \; u \in \mathrm{D}(T) \subset \mathrm{D}(A),$$

and (2) is more convenient to our purposes. Now let T be m-accretive and A be accretive. Then it is known that $T+A$ is also m-accretive if A is T-bounded, with $b<1$ (Cf. E. Nelson [4] and K. Gustafson [1] for Banach space case. See also T. Kato [2], p. 499 and I. Miyadera [3]). Our first result is concerned with the case that $b'=1$ in a Hilbert space (but which does not cover the case that $a \neq 0$, $b=1$). The second result is concerned with a kind of large perturbation. Since $-T$ generates a contraction semigroup if and only if T is m-accretive, these results are considered as a part of the perturbation theory for contraction semigroups. And our results might have some applications in the theory of partial differential equations as shown by the example below.

For the later use we recall here some properties of m-accretive operators in H. First we note that an m-accretive operator is accretive and densely defined (cf. [2], p. 279).

Lemma. *Let T be densely defined and accretive. In order that T have the m-accretive closure, it is necessary that the range $\mathrm{R}(T+\xi)$ of $T+\xi$ be dense in* H *for every $\xi>0$, and it is sufficient that this be true for some $\xi>0$.*

Proof. The necessity is clear by the definition of the m-accretive operator, so that we prove the sufficiency part. Since T is densely defined and accretive, T is closable (see [2], p. 268, Theorem 3.4).