

182. Finite Automorphism Groups of Restricted Formal Power Series Rings

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1. A formal power series $f = \sum_{i=0}^{\infty} a_i X^i$ with coefficients in a linearly topological ring A is called a restricted formal power series if the sequence of its coefficients $\{a_i\}$ converges to 0. All of such formal power series forms a subring of the formal power series ring $A[[X]]$, which is called a restricted formal power series ring and denoted by $A\{X\}$.

In [5], Samuel has obtained the following result:

Let A be a Noetherian complete local integral domain, and G a finite group consisting of A -automorphisms of $A[[X]]$. Then there exists a formal power series f such that the G -invariant subring of $A[[X]]$ is $A[[f]]$.

This is a generalization of the result of Lubin [2] which dealt with the case where A is the ring of p -adic integers and G is given by using a formal group law.

The main purpose of this paper is to prove the following:

Theorem. *Let A be a Noetherian complete integral domain with the maximal ideal \mathfrak{m} , and G a finite group consisting of A -automorphisms of $A\{X\}$. If the residue class field A/\mathfrak{m} is perfect, there exists a series $f \in A\{X\}$ such that the G -invariant subring $A\{X\}^G$ of $A\{X\}$ is $A\{f\}$.*

2. At first, we shall show some results concerning $A\{X\}$.

Lemma 1. *Let A be a linearly topological ring whose topology is complete and T_0 . Then, $A\{X+a\} = A\{X\}$ for any $a \in A$.*

Proof. For any $f = \sum_{i=0}^{\infty} a_i (X+a)^i \in A\{X+a\}$, we have $f = \sum_{i=0}^{\infty} b_i X^i$ in $A[[X]]$, where $\{b_i\}$ converges to 0. Hence, $f \in A\{X\}$.

If α is an ideal of A , by $\alpha\{X\}$ we denote the ideal of $A\{X\}$ consisting of all series $\sum_{i=0}^{\infty} a_i X^i$, $a_i \in \alpha$.

Lemma 2. *Let A be a linearly topological ring whose topology is complete and T_0 . Let \mathfrak{m} be a closed ideal of A such that every $m \in \mathfrak{m}$ is topologically nilpotent. If $f \in A\{X\}$ is a series such that $\bar{f} = f \bmod \mathfrak{m}\{X\}$ is a unitary polynomial with the degree $s \geq 1$, then $A\{X\}$ is the*