11. On Generalized Integrals. VI

Restrictions of (E.R.) Integral. I

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As it is already known, the set of all $(E.R.)$ integrable functions is very large. For example, it is proved, in studies of the A-integral (which coincides with the special $(E.R.)$ integral), that every continuous function whose product with any A-integrable function is A-integrable, is constant [1], and that in the set of all those A-integrable functions f for which the indefinite integral $A(x)=(A)\int_a^x f(x)dx$ is defined,[1] $A(x)$ cannot be the indefinite integral of only one function, to within a set of measure zero, i.e. there is no one-to-one correspondence between a function and its indefinite A-integral [8]. For this reason, there arose the problem of specialization of the A-integral and the $(E.R.)$ integral (see [5], [2], [7], [10], [4], [9]). On the other hand, in connection with the Denjoy integral defined as an extension of the Lebesgue integral, we have seen that for a function $f(x)$ Denjoy-integrable in the general sense, there exists some φ for which $f(x)$ is $(E.R.\ \varphi)$ integrable and both integrals are given as limit of the same approximating sums (see [3], V[2], Theorem 10). We now define, in this paper, the integrals, called $(E.R.\ \varphi)_2$ (resp. $(E.R.\ \varphi)_3$) integral, which are considered as specializations of Denjoy integral-type in the general (resp. special) sense of the $(E.R.\ \varphi)$ integral, and prove that a function $f(x)$ Denjoy-integrable in the general (resp. special) sense is also $(E.R.\ \varphi)_2$ (resp. $(E.R.\ \varphi)_3$) integrable for $\varphi(=\varphi(f))$ reasonably chosen, and both integrals coincide (Theorem 11).

We conserve the terminologies and the notation of the preceding papers I–V [6].

9. Restrictions of (E.R.) integrals (1). Let $\varphi(x)$ be a positive, Lebesgue-integrable function in a finite or infinite interval $[a, b]$. Denote the set of all measurable functions in $[a, b]$, by $\mathcal{M}$, or, for the purpose of calling special attention to the interval $[a, b]$, by $\mathcal{M}(a, b)$. Before defining integrals of the new sense, we first consider the following conditions instead of $[\gamma(\varphi)]$, where $[\gamma(\varphi)]$ is one of the

1) In general, the existence of the A-integral of $f(x)$ on $[a, b]$ does not imply its existence on $[c, d]\subseteq[a, b]$.

2) The reference number indicates the number of the Note.