6. *Applications of the Theorem Giving the Necessary and Sufficient Condition for the Normality of Product Spaces*

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We in this note intend to re-prove two well known results by means of Theorem in [1] furnishing the condition stated in the title. The first of the results suggests an explanation of Theorem from another point of view, and the second can be verified somewhat more shortly than the original proof.

Spaces in this note are Hausdorff, and notations and terminologies in [1] are used without referring.

Proposition 1 (C.H. Dowker [3]). *Let I be the real closed unit interval. If $X\times I$ is normal, then X is countably paracompact.*

Proof. It suffices ([3]) to show that for any decreasing sequence $\{F_i\}$ of closed sets of X with vacuous intersection we can find a sequence $\{G_i\}$ of open sets with vacuous intersection such that $F_i\subset G_i$ for all i. Let us denote $y_i=1/i$ for $i\neq 0$ and put

$$\begin{array}{ll} F_y=F_i & \text{for } y=y_i, \\ F_y=\emptyset & \text{otherwise;} \\ K_y=X & \text{for } y=0, \\ K_y=\emptyset & \text{otherwise.} \end{array}$$

Then we have for any $b\in I$

$$\limsup_b F_y \cap \limsup_b K_y=\emptyset$$

(we here use the assumption $\cap F_i=\emptyset$ for $b=0$), and, by Theorem in [1], there are families $\{G_y\subset X\,;\, y\in I\}$ and $\{H_y\subset X\,;\, y\in I\}$ with the properties

$$G_y\cap H_y=\emptyset,$$
$$G_b^0\supset c\text{-}\limsup_b G_y\supset F_b,$$
$$c\text{-}\limsup_b H_y\supset K_b.$$

Since $c\text{-}\limsup_0 H_y\supset X$, there is, for any $x\in X$, $V\in\mathfrak{N}_0$ such that

$$x\in\Big(\bigcap_{y\in V} H_y\Big)^0\subset H_y$$

for any $y\in V$. Take $y_i\in V$, then $x\in H_{y_i}$ and $x\notin G_{y_i}$. Consequently, we have

$$\bigcap_i G_{y_i}=\emptyset,$$
$$G_{y_i}^0\supset F_{y_i}=F_i.$$