31. *Some Characterizations of Strongly Paracompact Spaces*

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As is well known,

Theorem 1 (E. Michael [1]). *In a regular T_1-space X, the following properties are equivalent*:

(1) *Every open covering of X has a locally finite open covering as a refinement* (*i.e. X is paracompact*).

(2) *Every open covering of X has a locally finite closed covering as a refinement.*

In this paper, we will characterize the strongly paracompact spaces under the same fashion.

Let us recall the definitions of terms which are used in the statement of this paper. Let X be a topological space and $\mathfrak{A}$ be a collection of subsets of X. The collection $\mathfrak{A}$ is said to be *point finite* (resp. *point countable*) if every point of X is contained in at most finitely (resp. at most countably) many elements of $\mathfrak{A}$. $\mathfrak{A}$ is said to be *locally finite* (resp. *locally countable*) if every point x of X has the neighborhood which intersects only finitely (resp. only countably) many elements of $\mathfrak{A}$. $\mathfrak{A}$ is said to be *star finite* (resp. *star countable*) if every element of $\mathfrak{A}$ intersects only finitely (resp. only countably) many elements of $\mathfrak{A}$. X is said to be *paracompact* (resp. *strongly paracompact*) if every open covering of X has a locally finite (resp. star finite) open covering of X as a refinement.

Finally to state our results we need a next notion. Let $\{U_x \mid x \in X\}$ be a collection of subsets of X with the index set X, then its collection is *symmetric* if "$y \in U_x$" is equivalent to "$x \in U_y$".

We assume that all spaces in this paper are Hausdorff and, for any symmetric collection $\{U_x \mid x \in X\}$, U_x contains the point x for any point $x \in X$.

Theorem 2 (Yu. M. Smirnov [3]). *In a regular space X, the following properties are equivalent*:

(1) *Every open covering of X has a star finite open covering as a refinement* (*i.e. X is strongly paracompact*).

(2) *Every open covering of X has a star countable open covering as a refinement.*

By use of the above theorem, we shall prove the following theorem.