25. Sequential Convergence of Operators on Orlicz Spaces of Lebesgue-Bochner Measurable Functions in Various Operator Topologies and Some Applications*)

By J. DIESTEL
West Georgia College and Southwire Company,
Carrollton, Georgia, U. S. A.

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In an earlier paper [4], a complete set of representatives for the bounded, linear operators from an Orlicz space of Lebesgue-Bochner measurable functions to any Banach space was established. Presently, we are concerned with applying some of those results in order to establish criteria for the convergence of sequences of operators in various, frequently used operator topologies. Some of these results are extensions of some results in [1]; others are seemingly new even for the case considered therein.

We keep the same notation as in [4], and assume throughout that the generating convex function p satisfies the growth condition $p(2u) \leqslant cp(u)$ for some $c \geqslant 0$ and all $u \geqslant 0$.

Our first result entends those of [1] involving the convergence of a sequence of operators on the Orlicz space $L_p(v, Y)$ to the Banach space Z relative to the strong operator topology on $B(L_p(v, Y); Z)$. Recall that this topology is that of simple convergence in the terminology of [9]. Throughout we suppose that $T_n \in B(L_p(v, Y); Z)$ and $\mu_n \in M_q(V, B(Y; Z))$ are such that for $n \geqslant 0$:

$$T_n(f) = \int f d\mu_n$$

for all $f \in L_p(v, Y)$.

Theorem 1. *$T_n \to T_0$ (strongly) if and only if $\{\mu_n\}$ is bounded in $M_q(V, B(Y; Z))$ and for each $A \in V$, $\mu_n(A) \to \mu_0(A)$ (strongly) (relative to $B(Y; Z)$).*

Proof. Necessity follows from the inequality $\|\mu_n\|_{q,v} \leqslant 2\|T_n\|$ and the Banach-Steinhaus theorem.

To prove sufficiency, we note that the boundedness of $\{\mu_n\}$ and the inequality $\|T_n\| \leqslant \|\mu_n\|_{q,v}$ yields the boundedness of $\{\|T_n\|\}$. Thus it is enough that we show that $T_n g \to T_0 g$ for g belonging to some total family in $L_p(v, Y)$. By Theorem 8 of [3], the family $\{\chi_A y : A \in V, y \in Y\}$ is total in $L_p(v, Y)$; a simple calculation combined with the condition

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