67. *Characterizations of Strongly Regular Rings. II*

By S. LAJOS*) and F. SZÁSZ**)

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An associative ring A is called strongly regular if, for any element a of A, there exists an element x in A such that $a=a^2x$. Characterizations of strongly regular rings were given by Andrunakievič [1], Lajos [5], Luh [8], and Schein [9], moreover by Lajos and Szász [7]. This paper is connected with authors' earlier note [6]. For the semigroup theoretical terminology we refer to Clifford and Preston [2].

The purpose of this note is to give a further characterization of the class of strongly regular rings. For this aim we shall use four well known important propositions and in the proof of the theorem an equivalence relation discussed formerly by S. Lajos [3], which is a two-sided congruence relation on the multiplicative semigroup S of a strongly regular ring A.

Proposition 1. *A strongly regular ring A has no non-zero nilpotent elements.*

Proof. Obviously $a=a^2x$ implies $a^3x^2=a^2x=a$ and $a^{n+1}x^n=a$ where $a\in A$ and n is an arbitrary positive integer. Therefore $a^n=0$ implies $a=0$, for any element a of A.

Proposition 2. *Any idempotent element of a strongly regular ring A lies in the center of the ring.*

Proof. For $e=e^2$ and any $x\in A$ Proposition 1 and the relation

$$(exe-xe)^2=exexe-exexe-xexe+xexe=0$$

imply $exe=xe$. Similarly we have $exe=ex$, that is $ex=xe$.

Proposition 3. *Any strongly regular ring A is regular.*

Proof. Let a be an arbitrary element of A. Then $a=a^2x$ implies

$$(a-axa)^2=0.$$

Hence by Proposition 1, $a=axa$. Therefore $e=ax$ and $f=xa$ are idempotent elements and Proposition 2 implies $a=af=fa=axa=xa^2$.

Proposition 4. *Any strongly regular ring A is a two-sided ring.*

Proof. For any element a of A, the principal right ideal $(a)_R$ of A is a two-sided ideal because $a=a^2x=axa$, $(ax)^2=ax$,

$$(a)_R=(ax)_R=(axa)_R$$

and $yax=axy$ for any element y of A, by Proposition 2. Analogously can be proved that every principal left ideal of A is also two-sided.

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*) K. Marx University of Economics, Budapest, Hungary.

**) Math. Institute of the Hungarian Academy of Sciences, Budapest, Hungary.