61. On the Evolution Equations with Finite Propagation Speed

By Sigeru MIZOHATA

(Comm. by Kinjirô KUNUGI, M. J. A., March 12, 1970)

1. Introduction. Let

$$\left(\frac{\partial}{\partial t}\right)^m u(x, t)=\sum_{j<m} a_{\nu j}(x, t)\left(\frac{\partial}{\partial x}\right)^\nu\left(\frac{\partial}{\partial t}\right)^j u(x, t) \tag{1.1}$$

be an evolution equation defined on $(x, t) \in \boldsymbol{R}^l \times [0, T] \equiv \Omega$. We suppose all the coefficients are infinitely differentiable, and that for any time $t_0 \in [0, T)$ and any initial data

$$\left(\frac{\partial}{\partial t}\right)^j u(x, t_0)=\varphi_j(x) \in \mathcal{D} \ (j=0, 1, \cdots, m-1),$$

there exists a unique solution $u(x, t)$ for $t \in [t_0, T]$ in some functional space, say in $\mathcal{B}$ or in $\mathcal{D}_{L^p}$ $(1<p<+\infty)$.[1]

We say that (1.1) has a *finite propagation speed* if for any compact K in $\boldsymbol{R}^l$, there exists a finite $\lambda(K)$ (propagation speed) such that for any initial data $\Psi(x) \equiv (\varphi_0(x), \cdots, \varphi_{m-1}(x)) \in \mathcal{D}$, with initial time t_0, whose support is contained in K, the support of the solution $u(x, t)$ is contained in

$$\bigcup_{\xi \in \sup[\Psi]} (\xi, t_0)+C^+_{\lambda(K)},$$

where $C^+_{\lambda(K)}$ is the cone defined by $\{(x, t);\ |x| \leq \lambda(K)t,\ t \geq 0\}$.

We say that (1.1) is a *kowalevskian in* Ω, if the coefficients $a_{\nu j}(x,t)$ appearing in the second member are identically zero if $|\nu|+j>m$. Our result is the

Theorem. *In order that* (1.1) *have a finite propagation speed, it is necessary that* (1.1) *be kowalevskian in* Ω.

This theorem was proved by Gårding [1] in the case where all the coefficients are constant. Now we can prove this theorem by the same method as in [2]. The detailed proof will be given in a forthcoming paper. In this Note, to make clear our reasoning, we argue on a simple equation.

2. Localizations of equation. Let

$$\frac{\partial}{\partial t} u(x, t)=\sum_{|\nu| \leq p} a_\nu(x, t)\left(\frac{\partial}{\partial x}\right)^\nu u(x, t) \equiv a_p\left(x, t; \frac{\partial}{\partial x}\right) u(x, t) \tag{2.1}$$

be an evolution equation, *not kowalevskian*, in Ω. Without loss of generality, we may assume that at the origin the second member of (2.1) is effectively of order $p(>1)$. We can find then a complex num-

1) With regards to these notations, see [2]. As the proof given later shows, this conditions can be replaced by weaker conditions.