

60. On Axiom Systems of Ontology. I

By Shôtarô TANAKA

(Comm. by Kinjirô KUNUGI, M. J. A., March 12, 1970)

It is well known that Leśniewski's original system of ontology has the form of the following single axiom [1], [2]:

$$T. \quad a \varepsilon b \equiv [\exists c]\{c \varepsilon a\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}.$$

It is mentioned that the following expression can act as the single axiom of Ontology by C. Lejewski [1]:

$$A. \quad a \varepsilon b \equiv [\exists c]\{c \varepsilon a \wedge c \varepsilon b\} \wedge [c]\{c \varepsilon a \supset a \varepsilon c\}.$$

In this paper, we shall prove that T and A are equivalent. The proofs of theorems will be given in the form of suppositional proofs [1], [2].

Lemma 1. T implies A.

Proof.

$$T1. \quad a \varepsilon b \wedge b \varepsilon c \supset a \varepsilon c$$

$$\begin{array}{ll} \text{Proof.} & 1 \quad a \varepsilon b \\ & 2 \quad b \varepsilon c \supset \\ & 3 \quad [d]\{d \varepsilon b \supset d \varepsilon c\} \\ & 4 \quad a \varepsilon b \supset a \varepsilon c \\ & \quad a \varepsilon c \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{(premise)} \\ \\ \text{(T, 2)} \\ \text{(OI : 3)} \\ \text{(4, 1)} \end{array}$$

$$T2. \quad [\exists c]\{c \varepsilon a\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \\ \quad \quad \quad \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \supset a \varepsilon b \quad \text{(T)}$$

$$T3. \quad a \varepsilon b \supset a \varepsilon a$$

$$\begin{array}{ll} \text{Proof.} & 1 \quad a \varepsilon b \supset \\ & 2 \quad [\exists c]\{c \varepsilon a\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \\ & \quad \quad \quad \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \\ & 3 \quad [c]\{c \varepsilon a \supset c \varepsilon a\} \\ & \quad a \varepsilon a \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{(premise)} \\ \\ \text{(T, 1)} \\ \text{(} p \supset p \text{)} \\ \text{(T 2, 2, 3)} \end{array}$$

$$T4. \quad a \varepsilon b \wedge c \varepsilon a \supset a \varepsilon c$$

$$\begin{array}{ll} \text{Proof.} & 1 \quad a \varepsilon b \\ & 2 \quad c \varepsilon a \supset \\ & 3 \quad [de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\} \\ & 4 \quad a \varepsilon a \wedge c \varepsilon a \supset a \varepsilon c \\ & 5 \quad a \varepsilon a \\ & \quad a \varepsilon c \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{(premise)} \\ \\ \text{(T, 1)} \\ \text{(OI : 3)} \\ \text{(T 3, 1)} \\ \text{(4, 5, 2)} \end{array}$$

$$T5. \quad a \varepsilon b \supset [c]\{c \varepsilon a \supset a \varepsilon c\}$$

$$\begin{array}{ll} \text{Proof.} & 1 \quad a \varepsilon b \supset \\ & 2 \quad c \varepsilon a \supset a \varepsilon c \\ & \quad [c]\{c \varepsilon a \supset a \varepsilon c\} \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{(premise)} \\ \text{(T 4, 1)} \\ \text{(DI : 3)} \end{array}$$