53. Properties of Ergodic Affine Transformations of Locally Compact Groups. I

By Ryotaro SATO
Department of Mathematics, Josai University, Saitama

(Comm. by Kinjirô KUNUGI, M. J. A., March 12, 1970)

1. Introduction. Let G be a locally compact group. An affine transformation S of G is a one-to-one continuous transformation of G onto itself which is of the form $S(x)=a\cdot T(x)$, where a is an element of G and T is a continuous isomorphism of G onto itself. In his book, Lectures on Ergodic Theory [1], Halmos has raised a question: Can an automorphism of a locally compact but non-compact group be an ergodic measure preserving transformation? Recently Rajagopalan and Schreiber [3] have answered his question negatively, i.e., if G is a locally compact group which has an ergodic continuous automorphism with respect to a Haar measure on G then G is compact.

Then the following question has become of interest to the author: Can an affine transformation of a locally compact but non-compact group be an ergodic left Haar measure preserving transformation?

The aim of this paper is to study some properties of an ergodic affine transformation of a locally compact group and to give an answer to the above question. We shall prove the followings below:

(1) An affine transformation S of a locally compact group G which is not *bi*-continuous can not be ergodic with respect to a left Haar measure on G.

(2) An affine transformation S of a locally compact group G which is not left Haar measure preserving can not be ergodic with respect to a left Haar measure on G.

(3) If G is a locally compact totally disconnected non-discrete group which has an ergodic affine transformation S with respect to a left Haar measure on G then G is compact.

2. Properties of ergodic affine transformations.

Theorem 1. *Let G be a locally compact group with a left Haar measure μ. Suppose $S(x)=a\cdot T(x)$ is an affine transformation of G which is not bi-continuous. Then S is not ergodic with respect to μ.*

Proof. Since S is not *bi*-continuous, T is not *bi*-continuous. Thus there exists an open σ-compact subgroup H of G such that $T(H)\subset H$ and $T^{-1}(H)$ is not σ-compact by [2, Lemma 1].

Case I. Let there exist a positive integer n for which $S^{-n}(H)\supset H$. Let p be the smallest positive integer such that $S^{-p}(H)\supset H$. Then it