49. Notes on Finite Left Amenable Semigroups*)

By Takayuki TAMURA
University of California, Davis, California, U. S. A.

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Let S be a semigroup and $B(S)$ be the Banach space of all bounded complex or real valued functions on S. A semigroup S is called left [right] amenable if there is on $B(S)$ a mean m, that is, a linear functional m for which $\|m\|=1$ and $m(x)\geq 0$ if $x\geq 0$ on S and which is invariant under left [right] translations of elements of $B(S)$ by elements of S, in other words, $m(\alpha f)=m(f)$ where $(\alpha f)(x)=f(\alpha x)$, $f\in B(S)$, $x\in S$, α complex or real numbers, S is called amenable if S is left amenable and right amenable.

In (3I′), at p. 11 of [2] we can see the following proposition due to Rosen [5]:

Proposition 1. *A finite semigroup S is left amenable if and only if it has a unique minimal right ideal R. Then this right ideal is the union of the disjoint minimal left ideal $L_1, \cdots, L_k$ of S; each left ideal is a group, and all these groups are isomorphic. If u_i is the identity element of the group L_i, then $u_iu_j=u_j$ for all $i,j\leq k$, and if U is the set of these u_i, $R=L_i\times U$, and the left invariant means on S are supported on R and are exactly averaging over L_i crossed with arbitrary means on U.*

The statement concerning the minimal right ideal means that the right ideal is a right group [1], i.e. the direct product of a group and a right zero semigroup. Furthermore it is the kernel i.e. the minimal ideal. In this paper the author notices that a finite left amenable semigroup is characterized by left zero indecomposability of ideals.

By a left zero semigroup we mean a semigroup satisfying the identity $xy=x$. Every semigroup S has a smallest left zero congruence ρ_0, that is, ρ_0 is a congruence such that S/ρ_0 is a left zero semigroup, and ρ_0 is contained in all congruences ρ such that S/ρ are left zero semigroups. If ρ_0 is the universal relation, $\rho_0=S\times S$, then S is called left zero indecomposable. Refer undefined terminology to [1].

Theorem 2. *Let S be a finite semigroup. The following are equivalent:*

(1) *Every ideal of S is left zero indecomposable.*
(2) *The kernel K of S is a right group, $|K|\geq 1$.*
(3) *S has a unique minimal right ideal.*

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