76. On a Certain Type of Differential Hopf Algebras*)

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In [1] we have introduced strange differential Hopf structures arising from K-theory and have called them differential near Hopf algebras. One of the purposes of this paper is to find a general theory in order to make these strange differential Hopf algebras fit with the usual differential Hopf algebras.

Our main result is a generalization of a criterion of coprimitivity of Hopf algebras [5]. This enables us to use biprimitive form spectral sequences due to Browder [3] in researches of K-theory of H-spaces.

The detailed proofs will be published elsewhere.

1. By a G_2-module $M=M_0\oplus M_1$ we mean a Z_2-graded module over a field K. M has a canonical involution σ such that

$$\sigma|M_0=1 \quad \text{and} \quad \sigma|M_1=-1.$$

All algebraic structures such as algebras, coalgebras, differential algebras, etc., will be understood those over certain underlying G_2-modules [1]. In the present work, all algebras (or coalgebras) are equipped with augmentations and units (or counits), but are not necessarily associative.

Let M and N be differential G_2-modules. $M\otimes N$ is also a differential G_2-module. The usual switching map

$$T: M\otimes N\to N\otimes M$$

is an isomorphism of differential G_2-modules. Pick $\lambda\in K$. We define the *λ-modified switching* map

$$T_\lambda: M\otimes N\to N\otimes M$$

by $T_\lambda=(1+\lambda\cdot d\sigma\otimes d)T$. T_λ is also an isomorphism of differential G_2-modules and involutive, i.e., $T_\lambda^2=1$.

Generalizing the above T_λ, we can define the *λ-modified permutations* of tensor factors so that $\mathfrak{S}_n$ acts as a group of automorphisms of the differential G_2-module $M^{\otimes n}=M\otimes\cdots\otimes M$.

Our first basic idea is to replace the switching maps and permutations of tensor factors in the ordinary theory of Hopf algebras [5] by λ-modified ones and to construct a theory suitable to Hopf structures derived from mod p K-theory.

2. Let A and B be differential algebras (or coalgebras). Putting

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