100. *On a Ranked Vector Space*

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We show in this paper some relations between a ranked vector space and a linear topological space.

We suppose that a ranked vector space E satisfies the following conditions:

(M_1) Let E be a ranked vector space, a sequence, $\{u_n(x)\}$ any fundamental sequence of neighborhoods of an arbitrary point $x \in E$, and $v(x)$ any neighborhood of x (we denote this fact by $v(x) \in \mathfrak{V}(x)$), then there is a member $u_m(x)$ in $\{u_n(x)\}$ such that $u_m(x) \subset v(x)$.

Proposition 1. *Let E_1 and E_2 be two ranked vector spaces, and suppose that E_2 satisfies Condition* (M_1). *Let $f: E_1 \to E_2$ be continuous at a point $x \in E_1$, then for every neighborhood $v\{f(x)\}$ of the point $f(x) \in E_2$ there is a neighborhood $u(x)$ of the point $x \in E_1$ such that $f\{u(x)\} \subset v\{f(x)\}$.*

Proof. In order to show this, we proceed indirectly: i.e., assume that there is a neighborhood $v\{f(x)\}$ of the point $f(x)$ such that for any neighborhood $u(x)$ of x

$$f\{u(x)\} \not\subset v\{f(x)\}.$$

Let $\{u_n(x)\}$ be a fundamental sequence of neighborhoods of the point $x \in E_1$; i.e.,

$$u_0(x) \supset u_1(x) \supset u_2(x) \supset \cdots \supset u_n(x) \supset \cdots$$

and there is a sequence $\{\alpha_n\}$ of non-negative integers such that

$$\alpha_0 \leqq \alpha_1 \leqq \alpha_2 \leqq \cdots \leqq \alpha_n \leqq \cdots$$

where $\sup \{\alpha_n\} = \infty$, and for each n, $u_n(x) \in \mathfrak{V}_{\alpha_n}$. By assumption we have that for any n

$$f\{u_n(x)\} \not\subset v\{f(x)\},$$

i.e., for each n there is an element x_n in $u_n(x)$ such that $f(x_n) \notin v\{f(x)\}$. Hence, it follows from the definition of convergence that $\{\lim x_n\} \ni x$ and $f(x_n) \notin v\{f(x)\}$ for every n. Since $f: E_1 \to E_2$ is continuous at x, by the definition of continuity it follows that

$$\{\lim f(x_n)\} \ni f(x).$$

Hence there is a fundamental sequence $\{v_n\{f(x)\}\}$ such that

$$v_0\{f(x)\} \supset v_1\{f(x)\} \supset v_2\{f(x)\} \supset \cdots \supset v_n\{f(x)\} \supset \cdots$$

$$\beta_0 \leqq \beta_1 \leqq \beta_2 \leqq \cdots \leqq \beta_n \leqq \cdots$$

$$\sup \{\beta_n\} = \infty \text{ and for every } n$$

$$v_n\{f(x)\} \in \mathfrak{V}_{\beta n}, \qquad f(x_n) \in v_n\{f(x)\}.$$