99. On the Sets of Points in the Ranked Space. IV

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In this paper we report some properties holding in a sequentially compact ranked space.

We defined in previous papers the concepts such as r-open subsets [1], sequentially compact subsets [2], R-convergence and para-convergence of a sequence of points [3], $\{p_\alpha\}$, in a ranked space.

In the present paper we define a concept that a ranked space R is countably compact and a concept that R is totally bounded.

Definition 1. A ranked space is countably compact if and only if every countable open covering of the ranked space, S, has a finite subcovering of S.

Definition 2. A ranked space R is totally bounded if and only if for every natural number γ, there are suitable finite points of R, $a_1, a_2, \cdots, a_n$, and $V(a_i)$ $(i=1,2,\cdots,n)$ such that

(1) $\quad V(a_i)\in\mathfrak{V}_\gamma$ and $V(a_i)\cap V(a_j)=\emptyset$ $(i\neq j)$

and

(2) there does not exist any p of R and $V(p)\in\mathfrak{V}_\gamma$ satisfying

$$V(p)\subset R-\bigcup_{i=1}^{n}V(a_i).$$

Proposition 1. *If a ranked space R is sequentially compact, then R is countably compact.*

Proof. Suppose that R is not countably compact. Then, there is some countable open covering $\{U_i\}$ such that $\bigcup_{i=1}^{n}U_i\neq R$ for every natural number n. Hence there is a point a_n of R such that $a_n\in R-(\bigcup_{i=1}^{n}U_i)$ for every n. Since R is sequentially compact, the sequence $\{a_n\}$ has its subsequence $\{a_{m_\alpha}\}$ such that $a\in\{\lim_\alpha a_{m_\alpha}\}$. By the definition of R-convergence, there is a fundamental sequence $\{V_k(a)\}$ of neighborhoods of the point a for which there holds $a_{m_k}\in V_k(a)$.

On the other hand, since $a\in R$ and $\{U_i\}$ is a covering of R, there is an element U_l of $\{U_i\}$ such that $a\in U_l$. Since U_l is an r-open subset

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