# 98. *A Convergence Theorem in Measurable Function Spaces of Concave Type**)

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**1.** L. Schwarz has shown that in $L^p(\Omega\cdot\mu)$ $(0\leqq p<+\infty)$ for every $C$-sequence its sum is convergent [3]. In this note, we shall show that this fact is true in some type of measurable function spaces. Let $L$ be a measurable function space (topological vector space) with a linear topology $\mathcal{T}$. A sequence $f_n\in L(n=1,2,\cdots)$ is called *C-sequence* in $L$ if $\sum_{n=1}^{\infty}c_nf_n$ converges with respect to $\mathcal{T}$ for all sequences of real numbers $\{c_n\}$ which tend to 0.

Now, we shall consider some class of function spaces which includes $L^p(0\leqq p\leqq 1)$.

**2.** Let $\Omega$ be a measure space with measure $\mu$ where $\Omega$ is a union of mutually disjoint measurable set $A_\lambda(\lambda\in\Lambda)$ with finite measure and every measurable set of finite measure is contained in at most countable union from $A_\lambda(\lambda\in\Lambda)$. Let $\mathcal{M}$ be the set of all measurable functions.

Let $m$ be a functional on $\mathcal{M}$ with the following conditions.

(1) $0\leqq m(f)\leqq+\infty$ *for* $f\in\mathcal{M}$.

(2) $|f|\leqq|g|$ *a.e.* $\Rightarrow m(f)\leqq m(g)$.

(3) $m(f)=0$ *if and only if* $f=0$ *a.e.*

(4) $\inf(f,g)=0$ *i.e.* $f\cap g=0\Rightarrow m(f+g)=m(f)+m(g)$.

(5) $0\leqq f_n\uparrow,\ \sup_n m(f_n)<+\infty\Rightarrow m(f)=\sup_n m(f_n)$ *for* $f=\sup_n f_n$.

(6) $m(\alpha_nf)\to 0$ *as* $\alpha_n\to 0$ *for every* $f$ *with* $m(f)<+\infty$.

(7) $m(\alpha f)\geqq\alpha m(f)$ *for* $1\geqq\alpha\geqq 0$.

(8) $m(\chi_E)<+\infty$ *for every characteristic function* $\chi_E$ *of* $E$ *with* $\mu(E)<+\infty$.

We shall consider a subset of $\mathcal{M}$: $L_m=\{f\in\mathcal{M},\ m(f)<+\infty\}$. We shall identify $f$ and $g$ if $f=g$ a.e. in $L_m$. If $m(f)=\int|f|^pd\mu(0<p\leqq 1)$, then $L_m$ coincides with $L^p$. If $\Omega=\bigcup_{i=1}^{\infty}A_i$ (disjoint union) $(0<\mu(A_i)<+\infty$ for all $i=1,2,\cdots)$ and $m(f)=\sum_{i=1}^{\infty}\frac{1}{2^i\mu(A_i)}\int_{A_i}\frac{|f|}{1+|f|}d\mu$, then $L_m$ is the space of all measurable functions (essentially finite). In this case, (5) must be changed.

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*) Dedicated to Professor Hidegoro Nakano on his 60th birthday.