94. On Representations of Tensor Products of Involutive Banach Algebras

By Takateru OKAYASU
College of General Education, Tôhoku University, Sendai

(Comm. by Kinjirô KUNUGI, M. J. A., May 12, 1970)

In this paper we want to give a complementary result to the previous paper [2]. The aimed theorem states that; if each of involutive Banach algebras A and B has approximating identity and has a faithful representation, then any representation π of the algebraic tensor product $A\odot B$ of A and B is *subcross* in the sense of [1], in other words, it is satisfied that

$$\|\pi(x\otimes y)\|\leqq\|x\|\,\|y\| \qquad \text{for } x\in A \text{ and } y\in B.$$

This means that

$$\|\pi(t)\|\leqq\|t\|_\gamma \qquad \text{for } t\in A\odot B,$$

where $\|\ \ \|_\gamma$ denotes the γ-norm on $A\odot B$, and that there are representations π^1, π^2 of A, B, called the *restrictions* of π on A, B, respectively, on the representation space of π such that

$$\pi(x\otimes y)=\pi^1(x)\pi^2(y)=\pi^2(y)\pi^1(x) \qquad \text{for } x\in A \text{ and } y\in B.$$

These assertions seem to be important in investigations of algebraic tensor products of involutive Banach algebras from standpoints of C^*-algebras.

As Professor A. Guichardet kindly pointed out by his private letter, the arguments of Lemma 1 and Theorem 1 in [2] are lacking in exactness. A part of the following is devoted to remove their inexactness. It is done by imposing a natural condition upon involutive Banach algebras considered. The author wishes to take this opportunity to deeply thank Professor Guichardet.

1. Preliminaries. An involutive algebra means an algebra over the complex number field C with an involution always denoted by *. Given an involutive algebra A, the adjunction A_1 of the identity to A means A itself when A has an identity, the involutive algebra of all formal sums $u=x+\lambda$ of $x\in A$ and $\lambda\in C$ when A has no identity. A representation of an involutive algebra means its involution-preserving homomorphism into the algebra of bounded linear operators on a complex Hilbert space.

An involutive Banach algebra means an involutive algebra equipped with a norm under which it is a Banach algebra and its involution is isometric. When A is an involutive Banach algebra, as