123. *Some Remarks on the Approximation of Nonlinear Semi-groups*

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1. Let X be a Banach space and U be a subset of X. Let $\{T(t)\,;\,t\geq 0\}$ be a family of nonlinear operators from U into itself satisfying the conditions:

(i) $T(0)=I$ (the identity mapping) and $T(t+s)=T(t)T(s)$ for $t, s\geq 0$.

(ii) For $x\in U$, $T(t)x$ is strongly continuous in $t\geq 0$.

(iii) $\|T(t)x-T(t)y\|\leq\|x-y\|$ for $x, y\in U$ and $t\geq 0$.

Such a family $\{T(t)\,;\,t\geq 0\}$ is called a nonlinear contraction semi-group on U. We define the infinitesimal generator A of the semi-group $\{T(t)\,;\,t\geq 0\}$ by

$$Ax-\lim_{h\to 0+}h^{-1}(T(h)-I)x$$

and the weak infinitesimal generator A' by

$$A'x=w\text{-}\lim_{h\to 0+}h^{-1}(T(h)-I)x$$

if the right sides exist. (The notation "lim" ("w-lim") means the strong limit (the weak limit) in X. We denote the domain of A by $D(A)$.)

H. F. Trotter [6] established the following result for linear contraction semi-groups.

Theorem. *Suppose that $\{T(t)\,;\,t\geq 0\}$ and $\{T'(t)\,;t\geq 0\}$ are linear contraction semi-groups of class (C_0) in the Banach space X with infinitesimal generators A and B, respectively. If $A+B$ (or its closure) is the infinitesimal generator of a semi-group $\{S(t)\,;\,t\geq 0\}$ of class (C_0), then*

$$S(t)x=\lim_{h\to 0+}(T(h)T'(h))^{[t/h]}x,\ x\in X.$$

[ ] denotes the Gaussian bracket.

In Section 2, we shall prove an extension of this theorem for the case of nonlinear contraction semi-groups on a subset U of a Banach space X. In Section 3, we shall approximate the semi-group $\{S(t)\,;\,t\geq 0\}$ by using $2^{-1}(T(2h)+T'(2h))$ which is the arithmetic mean of $T(2h)$ and $T'(2h)$. Note that, roughly speaking, $T(h)T'(h)$ may be regarded as the geometric mean of $T(2h)$ and $T'(2h)$.

2. The proofs in this paper are based upon the following theorem which was proved by I. Miyadera and S. Oharu [3], [4].