122. *On Quasi-Souslin Space and Closed Graph Theorem*

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L. Schwartz defined *Souslin space* as any continuous image of a complete separable metric space and a generalized closed graph theorem is obtained in [1] and [2] for this class of spaces. Here we consider a slightly wider class of topological spaces, namely *quasi-Souslin spaces*, and prove a closed graph theorem extending the method in [2].

A filter Φ is said to be *S-filter* if Φ has a countable basis $\{S_n\}$ such that $\bigcap_n S_n=\phi$.

A Housdorff topological space E is called a *quasi-Souslin space*, if there exists a sequence of S-filters Φ_n $(n=1,2,\cdots)$ such that every ultrafilter Ψ with $\Psi \not\supset \Phi_n$ $(n=1,2,\cdots)$ converges in E. In the sequel Φ_n are called *defining filters for E.*

Let A be a subset of a set E and Φ a filter in E. We say that A is *disjoint from* Φ if there is B in Φ such that $A\cap B=\phi$. If A is not disjoint from Φ, we denote the filter $\{A\cap B\,|\,B\in\Phi\}$ in A by Φ_A. We identify the filter Φ_A in A with the filter Φ in E if $A\in\Phi$. Let φ be a mapping from a set E into a set F and Φ,Ψ filters in E,F respectively. $\varphi(\Phi)$, the *image of* Φ *by* φ, is defined as the filter generated by $\{\varphi(A)\,|\,A\in\Phi\}$. When $\varphi(E)$ is not disjoint from Ψ, $\varphi^{-1}(\Psi)$, the *inverse image of* Ψ *by* φ is defined as the filter generated by $\{\varphi^{-1}(A)\,|\,A\in\Psi\}$.

A subset A of topological space E is said to be *everywhere second category in E*, if any non-void intersection $U\cap A$ with an open set U in E is second category. As well known, if A is second category, the set $O(A)$ of all the elemets x in E for which $A\cap V$ is second category for every neighbourhood V of x is not empty and $O(A)\cap A$ is everywhere second category in E.

First we show that the class of quasi-Souslin spaces, as in the case of Souslin spaces, is closed by the following operations:

(1) *The image* $E=\varphi(F)$ *of a quasi-Souslin space* F *by a continuous mapping* φ *is quasi-Souslin.*

(2) *The closed subspace* E *of a quasi-Souslin space* F *is quasi-Souslin.*

(3) *The product space* $E=\prod_n E_n$ *of quasi-Souslin spaces* E_n $(n=1,2,\cdots)$ *is quasi-Souslin.*

(4) *The inductive limit* E *of quasi-Souslin spaces* E_n $(n=1,2,\cdots)$ *is quasi-Souslin.*