118. *On a Theorem of E. Michael and K. Nagami*

By Masao SAKAI

(Comm. by Kinjirô KUNUGI, M. J. A., June 12, 1970)

E. Michael [1] and K. Nagami [2] proved the following theorem.

Theorem. *Every metacompact and collectionwise normal space is paracompact.*

A space is called metacompact if every open covering of it can be refined by a point-finite open covering.

We shall prove this theorem, using the following lemmas for point-finite open covering of a topological space.

Lemma 1. *Every point-finite open covering* $\{U_\alpha\}_{\alpha\in A}$ *of a topological space contains an irreducible subcovering* (see [3]).

Lemma 2. *For each point-finite open covering* $\{U_\alpha\}_{\alpha\in A}$ *of a normal space, there exists an open covering* $\{V_\alpha\}_{\alpha\in A}$ *of the space such that* $\bar{V}_\alpha\subset U_\alpha$ *for every* $\alpha\in A$ (see [4]).

Proof of Theorem. 1. Let X be a meta-compact and collectionwise normal space, and $\boldsymbol{C}$ be an any open covering of X. Then, there exists a point-finite open refinement $\boldsymbol{U}=\{U_\alpha\}_{\alpha\in A}$ of the covering $\boldsymbol{C}$. By Lemma 1, we can assume that the covering $\boldsymbol{U}$ is irreducible. We are going to prove that the open covering $\boldsymbol{U}$ has a σ-discrete open refinement $\{\boldsymbol{W}_n\}_{n=1,2,3,\cdots}$, $\boldsymbol{W}_n=\{W_{n,\beta}\}_{\beta\in B_n}$. Then, the space X will be paracompact.

2. By Lemma 2, for the irreducible open covering $\{U_\alpha\}_{\alpha\in A}$, there exists an open refinement $\{V_\alpha\}_{\alpha\in A}$ such that $\bar{V}_\alpha\subset U_\alpha$, for every $\alpha\in A$. *Put* $F_\alpha=\bar{V}_\alpha-\bigcup_{\substack{\alpha'\in A\\ \alpha'\neq\alpha}}U_{\alpha'}$, *then* $F_\alpha\neq\phi$, *for every* $\alpha\in A$.

For, if $F_\alpha=\phi$, then $\bigcup_{\substack{\alpha'\in A\\ \alpha'\neq\alpha}}U_{\alpha'}\supset\bar{V}_\alpha\supset V_\alpha$. By $U_{\alpha'}\supset V_{\alpha'}$,

$$\bigcup_{\substack{\alpha'\in A\\ \alpha'\neq\alpha}}U_{\alpha'}\supset V_\alpha\cup\bigcup_{\substack{\alpha'\in A\\ \alpha'\neq\alpha}}V_{\alpha'}=X.$$

This contradicts to the irreducibility of covering $\{U_\alpha\}_{\alpha\in A}$. Then $F_\alpha\neq\phi$.

3. *Any point* x *of the space* X *which is contained in only one* U_α *of the family* $\{U_\alpha\}_{\alpha\in A}$, *is contained in* F_α.

For, suppose that $x\in U_\alpha$ and $x\not\in U_{\alpha'}$ ($\alpha'\in A, \alpha'\neq\alpha$). By $\bar{V}_{\alpha'}\subset U_{\alpha'}$, $x\not\in V_{\alpha'}$. As the family $\{V_\alpha\}_{\alpha\in A}$ is a covering of X, $x\in V_\alpha\subset\bar{V}_\alpha$. Then, $x\in\bar{V}_\alpha-\bigcup_{\substack{\alpha'\in A\\ \alpha'\neq\alpha}}U_{\alpha'}=F_\alpha$.

4. *The family* $\{F_\alpha\}_{\alpha\in A}$ *is a closed discrete family.*

For, any point x of X is contained in some U_α. For a neighborhood of x, put $V(x)=U_\alpha$.